

Joint Admission Control, Channel Assignment and QoS Routing for Coverage Optimization in Multi-hop Cognitive Radio Cellular Networks

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Abstract—In recent years, cognitive radio technology (CR) has been proposed to allow unlicensed secondary users (SUs) to opportunistically access the channels unused by primary users. As a result, there is a lot of recent interests on studying cognitive radio cellular networks (CogCells) that can support both PUs and SUs. Due to the limited transmission range of SUs, in this work we consider supporting Multi-hop infrastructure-based secondary systems (SSs), where SUs can communicate with the BS over multiple hops. The use of SSs improves the reliability and coverage compared to its single-hop counterpart. In addition, SUs are allowed to access multiple channels, which helps to increase transmission reliability and coverage and relieve interference at PUs. To enable multi-hop secondary transmissions, it is also important to support efficient routing. In CogCells, efficient admission control, channel assignment and routing is crucial for the coverage optimization of SSs and to ensure the QoS requirements in CogCells.

In this paper, we mathematically formulate the problem of joint admission control, channel assignment and QoS routing to maximize the coverage of SUs in a CogCell system that supports multi-hop secondary transmissions, taking into account the interference constraints and QoS requirements from the PUs and admitted SUs. To our best knowledge, this is the first study that attempts to optimize the coverage of SUs in multi-hop CogCells with the concurrent support of the above three important procedures. We show that the problem is NP-hard and propose three different algorithms to solve the coverage optimization problem and give the theoretical analyses of its performances in terms of approximation ratio to the optimum. Our solutions include a greedy heuristic approximation scheme, an algorithm that can provide exact solution, and a new approximation solution with a poly-logarithmic approximation ratio guarantee, e.g., the performance of our algorithm is within a poly-logarithmic factor of that of any optimal algorithm for the problem. Our preliminary simulation results indicate that our new approximation algorithms can effectively exploit the increased number of SUs and channels, and performs much better than the theoretical worst case bound.

I. INTRODUCTION

Recent studies show that the licensed primary wireless users (PUs) rarely utilize all the assigned frequency channels at any time and location, therefore leading to many spectrum holes. In order to improve spectrum usage efficiency, cognitive radio technology (CR) has been proposed in recent years to allow

unlicensed secondary users (SUs) to transmit opportunistically over the unused spectrum without interrupting the operation of PUs. CR shows a great promise to enhance the spectrum utilization efficiency [5]. This has raised a lot of recent interests on studying cognitive radio cellular networks (CogCell), where a CogCell consists of one base station (BS), a set of primary users (PUs) and secondary users (SUs), and a group of channels to be used.

Due to high density, spectrum sharing with PUs, mobility of SUs, the interference constraints and the QoS requirements of PUs, not all SUs can be admitted to the system by the operator. As an SU may have a lower transmission power, in order to provide a better coverage for SUs thus making more revenue from secondary transmissions, in this work we consider a model that allows SUs to communicate with the BS over multiple hops. That is, each admitted SU operates not only as a host, but also as a router to forward packets on behalf of other admitted SUs that may not be within the direct transmission range of the BS. Moreover, the admitted SUs are also supposed to reach certain quality of service (QoS) such that the maximal time delay from any admitted SUs to the BS should be bounded in an acceptable threshold. In a CogCell, spectrum sharing between PUs and SUs may significantly affect the quality of service (QoS) of the CogCell. If an SU shares the same channel as a PU, it will increase the interference power at the receiver of the PU, thus decreasing its signal-to-interference-plus-noise-ratio (SINR). On the other hand, the transmission of an SU will not affect the QoS of PUs operating on different channels. Thus, allowing SUs to access multiple channels can potentially offer increased reliability and coverage, and reduce the interference at PUs.

From the above discussions, we can see that effective admission control is needed to enlarge the coverage of CogCells and ensure the QoS requirements of both PUs and admitted SUs. It is also important to develop efficient channel assignment schemes to greatly reduce the interference effect of nearby transmissions. Finally, to support multi-hop transmissions of SUs, the routing scheme should effectively alleviate potential congestion to the base station and improve the system throughput. Therefore, efficient admission control, channel assignment and QoS routing are very crucial for the coverage optimization

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of SSs and to ensure the QoS requirements in CogCells. It also highlights the necessity and importance to investigate cross-layer optimization problem in CogCell systems. Although very important, this joint optimization problem has not been investigated in a CogCell system that supports multi-hop secondary transmissions.

Joint admission control, channel assignment, and power allocation problem for maximization of secondary revenue in the CogCell has not been explored well so far even in a single-hop scenario. Most of work only focused on the single-channel scheme. Islam et al. [7] investigated the distributed scheme in the CogCell with one BS equipped with multiple antennas, one primary transmitter (PT) and one primary receiver (PR). In [12] and [13], Xing et al. proposed a distributed constrained power control algorithm under the consideration of the CogCell with one PR, and several SUs with separate secondary receivers, based on a game-theory approach. In [15], Zhang et al. proposed a minimal SINR removal algorithm (MSRA) for the CogCell with one PU. The NP-hardness of this problem had also been showed in [15]. A power control scheme had been proposed in [16] for the CogCell under a strong assumption that all SUs are admitted to access the channel to BS. In [11], Xiang et al. introduced three QoS-aware admission and power control schemes which included an exact solution based on dynamic programming, a greedy heuristic algorithm and a minimal SINR removal algorithm which is a simple extension of MSRA from [15]. Very recently, a new joint scheme that takes into account QoS-aware admission control, channel assignment and power allocation scheme was proposed by xin [10] and it had been shown that the secondary revenue achieved by the proposed approximation algorithm is only a logarithmical factor away from the optimum. However, these work only considered the single-hop scenario, which is much simpler than our problem and does not need to consider routing strategies.

In this paper, we mathematically formulate the problem of joint admission control, channel assignment and QoS routing to maximize the coverage of BS for SUs in the multi-hop infrastructure-based SS of CogCell, taking into account the interference constraints and QoS requirements from both PUs and admitted SUs. **To our best knowledge, this is the first effort of studying the coverage optimization problem with concurrent consideration of the three important procedures in a CogCell system supporting multi-hop secondary transmissions.** We show that the problem considered is NP-hard, and propose three different algorithms to solve the coverage optimization problem and show the theoretical analyses of its performances in terms of approximation ratio to the optimum. Our solutions include a greedy heuristic approximation scheme, an algorithm to look for exact solution, and more importantly a new approximation solution with a poly-logarithmic approximation ratio guarantee. The performance of our approximate algorithm is within a poly-logarithmic factor of that of any optimal algorithm for the problem. Our preliminary simulation results indicate that our new approximation algorithm can effectively support an in-

creased number of SUs, in a CogCell system that allows SUs to access multiple channels and communicate with the BS over multiple hops, and it performs much better than the theoretical worst case bound. It is worth mentioning that our scheme can be extended to the adaptive power control scenario. **Due to the space constraint, we defer the power control related issues and performance evaluations of our new approximation algorithms to the full version of this paper.**

The rest of the paper is organized as follows. In Section II, we introduce the network model and system architecture of CogCells, the model of wireless interference, and the QoS metrics for PUs and admitted SUs. We formulate the coverage optimization problem and show its NP-hardness in Section III. In Section IV, we present our schemes for joint admission control, channel assignment, and QoS routing. Finally, we conclude the paper in Section V.

II. SYSTEM MODEL AND ASSUMPTIONS

In this section, we will first introduce the network model and system architecture used in this work for CogCells. Then we describe the model of wireless interferences caused by PUs and admitted SUs and the definition of SINR. Finally, we give the metrics of the QoS requirements which have to be guaranteed and provided to PUs and admitted SUs in the CogCell system which allows SUs to perform multi-hop transmissions.

A. Network Model and System Architecture

In the literature, the models used in CogCells for spectrum sharing between PUs and SUs can be cataloged into two main classes. The first one is called overlay model where SUs should stop transmission on the channels they currently occupy and switch to another unused channel as long as PUs are detected to be using these channels. Another one is underlay model in which the SUs and PUs can coexist and share the same spectrum with each other by employing Code Division Multiple Access (CDMA) as long as the interference caused by the SUs to PUs is less than the predefined system threshold [2], [17], [11], [10]. In this paper, we concentrate on the underlay model. In a CogCell, the system consists of one BS, a set of PUs and SUs, and a group of channels which can be used in the system. However, any admitted SUs or PUs can be only allowed to use at most one channel at any time due to hardware constraints at each SU or PU that is the exactly same assumption used in [10], [11]. Each admitted SU operates not only as a host, but also as a router to forward packets on behalf of other admitted SUs that may not be within the direct transmission range of the BS. Therefore, multi-hop infrastructure-based secondary systems can enhance the reliability and improve the coverage of the BS compared to its single-hop counterpart. It is worth mentioning that this is the first time that multi-hop SSs in CogCell systems are investigated. In general, a PU can be in one of two modes, transmitting or receiving, depending on if the PU is transmitting or receiving data from the BS. Without the loss of generality, the PUs in the CogCell can be represented by the

PTs where the PUs are transmitting (or planning to transmit) data on some channels, and the PRs where the PUs stay in the receiving mode. Another motivation of such a division on PUs is to guarantee different QoS requirements which will be described later. Figure 1 shows the system model of the CogCell used in this paper.

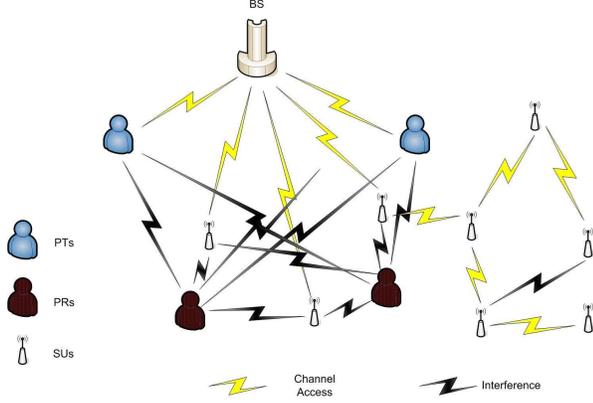


Fig. 1. An example of the CogCell.

To simplify our presentation and clarify the novelty and significance of our work, we use the similar notations as ones used in [10], but we concentrate on the scenario where multi-hop secondary systems are supported. Let N_s, N_p^t, N_p^r and N_w denote the sets of SUs, PTs, PRs, and the available channels respectively. The number of SUs, PTs, PRs, and channels can be denoted by n_s, n_p^t, n_p^r and n_w respectively, where $n_s = |N_s|, n_p^t = |N_p^t|, n_p^r = |N_p^r|$, and $n_w = |N_w|$. Moreover, let $N_p^t(w), N_p^r(w), n_p^t(w), n_p^r(w)$ denote the sets of PTs and PRs which are accessing the channel w and the cardinalities of the corresponding sets respectively. In a CogCell system, the operator of the BS can control the admission of SUs into service while maximizing the number of admitted SUs in order to enlarge the coverage of the BS for SUs. It is clear that the connectivity-related issues should be also taken into account by the operator. Moreover, the QoS (e.g., the minimum data transmission rate) required by any admitted SUs should be also guaranteed. However, each SU i will also generate the signal and interference power τ_{ij} at PR j if it is allowed to access the same channel as PR j . Since the PUs have a higher privilege than the SUs in the CogCell, the signal and interference power caused by all admitted SUs and all PTs to any PR j can not exceed the predefined system threshold Γ_j . In order to maximize the coverage of a secondary system, a proper subset of the SUs (e.g., admitted SUs) need to be selected to access the channels allowed to communicate with the BS in a multi-hop manner under an acceptable interference threshold at each PR. Moreover, the system also needs to guarantee and provide the QoS to PTs and admitted SUs according to their individual requirements. The detailed QoS requirements from PTs and admitted SUs to the system will be introduced in the following sections. For the convenience of presentation, we list all notations and symbols used in this paper in Table I.

B. Wireless Transmission and Interference Model

When SUs and PTs transmit over the same channel as the one used by PRs or SRs, PRs and SRs will receive the interference power from all these admitted SUs and PTs. Note that we assume that admitted SUs can be in both transmission and receiving modes in this work. Therefore, we should take into account the interference effects made by all admitted SUs to the system. Different types of interference models have been studied in the literature, which include physical interference model (PhyIM) [4], [10], [11], [15], [16], fixed protocol interference model (fPrIM) [9], RTS/CTS model (RTS-CTS) [1], and transmitter interference model (TxIM) [14]. In this paper, we adopt the PhyIM. Let $R_T(\mu)$ and $R_I(\mu)$ denote the transmission range and the interference range of user μ in the CogCell system respectively, where $\mu \in N_s \cup N_p^t \cup N_p^r$. Typically, $R_T(\mu) < R_I(\mu) \leq cR_T(\mu)$ for some constant $c > 1$. Normally, we call the ratio $\gamma_\mu = \frac{R_I(\mu)}{R_T(\mu)}$ as Interference-Transmission ratio for user μ , where $1 \leq \gamma_\mu \leq 5$ in practice. If we do not specify the interference at a user μ in CogCell system, we always refer to the interference caused by all users who have μ within their interference range. Specifically, let $\mathfrak{T}(\mu)$ and $\mathfrak{J}(\mu)$ denote the subsets of users which contain the PUs and SUs within the transmission range and the interference range of user μ respectively. Furthermore, let $\tau_{i,j}^w$ and $\zeta_{k,j}^w$ denote the interference power received at PR j due to the transmission from SU $i \in N_s$ and PT $k \in N_p^t$ on channel w respectively. According to the PhyIM, $\tau_{i,j}^w$ and $\zeta_{k,j}^w$ can be expressed as follows.

$$\tau_{i,j}^w = h_{ij}^{sp}(w)P_i^s(w), \forall i \in N_s \cap \mathfrak{J}(j), \forall j \in N_p^r \quad (1)$$

and

$$\zeta_{k,j}^w = h_{kj}^{pp}(w)P_k^p(w), \forall k \in N_p^t \cap \mathfrak{J}(j), \forall j \in N_p^r \quad (2)$$

where $P_i^s(w)$, $P_k^p(w)$, $h_{ij}^{sp}(w)$ and $h_{kj}^{pp}(w)$ denote the transmission powers on channel w at SU i and PT k , and the power attenuation from SU i , PT k to the PR j respectively. From the PhyIM, $h_{ij}^{sp}(w)$ and $h_{kj}^{pp}(w)$ can be calculated as follows.

$$h_{ij}^{sp}(w) = \frac{G_i^s(w)G_j^{pr}(w)}{(d_{ij}^{sp})^\alpha}, \forall i \in N_s \cap \mathfrak{J}(j), \forall j \in N_p^r \quad (3)$$

and

$$h_{kj}^{pp}(w) = \frac{G_k^{pt}(w)G_j^{pr}(w)}{(d_{kj}^{pp})^\alpha}, \forall k \in N_p^t \cap \mathfrak{J}(j), \forall j \in N_p^r \quad (4)$$

where $G_i^s(w)$, $G_k^{pt}(w)$, $G_j^{pr}(w)$, d_{ij}^{sp} , d_{kj}^{pp} , and α denote the antenna gains of SU i , PT k and PR j on channel w , the distances from the SU i and PT k to the PR j , and the path fading factor respectively.

Consequently, the signal plus interference power $T_j^I(w)$ accumulated at PR j on channel w due to the transmission from all admitted SUs and PTs with exactly the same channel w can be formulated as follows.

$$T_j^I(w) = \sum_{i=1}^{n_s} \tau_{ij}^w x_{iw} + \sum_{k=1}^{n_p^t} \zeta_{k,j}^w$$

$$\begin{aligned}
&= \sum_{i=1}^{n_s} \frac{G_i^s(w)G_j^{pr}(w)P_i^s(w)x_i}{(d_{ij}^{sp})^\alpha} + \sum_{k=1}^{n_p} \frac{G_k^{pt}(w)G_j^{pr}(w)P_k^p(w)}{(d_{kj}^{pp})^\alpha} \\
&\leq \Gamma_j^w, \forall j \in N_p^r
\end{aligned} \tag{5}$$

where x_{iw} is a binary variable where $x_{iw} = 1$ indicates SU i is admitted to transmit to the BS on channel w , otherwise SU i is forbidden to transmit on channel w in the CogCell. Due to the privileges of PUs and coexistence regulation in underlay model [11], [15], $T_j^I(w)$ can not exceed the predefined system threshold Γ_j^w on channel w at PR j . Note that the interference model used in [11], [15] is not reliable since the interference limitation at PR j does not take account into the accumulated interference from PTs. Without loss of the generality, we always assume that

$$\sum_{k=1}^{n_p} \zeta_{kj} \leq \sum_{k=1}^{n_p} \frac{G_k^{pt}(w)G_j^{pr}(w)P_{max}^p(w)}{(d_{kj}^{pp})^\alpha} \leq \Gamma_j^w, \forall j \in N_p^r. \tag{6}$$

Similarly, we can also define the corresponding terminologies of interference effect at secondary receivers (SRs) as follows. The interference power $T_q^I(w)$ accumulated at SU q on channel w due to the transmission from all admitted SUs and PTs with exactly the same channel w can be formulated by

$$\begin{aligned}
T_q^I(w) &= \sum_{i=1}^{n_s} \tau_{iq}^w x_{iw} + \sum_{k=1}^{n_p} \zeta_{kq}^w \\
&= \sum_{i=1}^{n_s} \frac{G_i^s(w)G_q^s(w)P_i^s(w)x_{iw}}{(d_{iq}^{ss})^\alpha} + \sum_{k=1}^{n_p} \frac{G_k^{pt}(w)G_q^s(w)P_k^p(w)}{(d_{kq}^{ps})^\alpha} \\
&\leq \Gamma_q^w, \forall q \in N_s \wedge i \in \mathcal{I}(q) \wedge k \in \mathcal{I}(q).
\end{aligned} \tag{7}$$

C. SINR Definition

According to the description of PhyIM in [10], the SINR of PT k on channel w can be expressed as follows.

$$\xi_k^p(w) = \frac{h_k^{pb}(w)P_k^p(w)}{N_0 + I_s(w) + I_p(w) - h_k^{pb}(w)P_k^p(w)}, \forall k \tag{8}$$

where $h_k^{pb}(w)$ and $P_k^p(w)$ denote the power attenuation to the BS and the transmission power of PT k on channel w respectively. Moreover, N_0 represents the background noise received at the BS, $I_s(w)$ and $I_p(w)$ present the interferences received from all admitted SUs and PTs on channel w respectively. According to the definition of SINR, $I_s(w)$ and $I_p(w)$ can be defined as follows.

$$I_s(w) = \sum_{i=1}^{n_s} h_i^{sb}(w)P_i^s(w)x_{iw} \tag{9}$$

$$I_p(w) = \sum_{k=1}^{n_p} h_k^{pb}(w)P_k^p(w) \tag{10}$$

where $h_i^{sb}(w)$ and $h_k^{pb}(w)$ denote the power attenuation on the channel w from SU i and PT k to the BS respectively.

By the definition of SINR, we can also have

$$h_i^{sb}(w) = \frac{G_i^s(w)G^b(w)}{(d_i^{sb})^\alpha}, \forall i \in N_s \tag{11}$$

and

$$h_k^{pb}(w) = \frac{G_k^{pt}(w)G^b(w)}{(d_k^{pb})^\alpha}, \forall k \in N_p^t \tag{12}$$

where d_i^{sb} , d_k^{pb} , $G_k^{pt}(w)$ and $G^b(w)$ denote the distances from SU i and PT k to the BS and the antenna gains of PT k and the BS on the channel w respectively.

Due to the privilege and the QoS requirements at PUs, we assume that all PUs can communicate with BS within one hop. In this work, we mainly focus on supporting coverage extension of BS to allow SUs to communicate with the BS in a multi-hop fashion. Let $hop(i)$ denote the hop distance from the SU i to BS in SSs of the CogCell system, e.g., the subnetwork induced only by SUs. Similarly as Equation 8, the SINR of the admitted SU i that has $hop(i) = 1$ can be given by

$$\xi_i^s(w) = \frac{h_i^{sb}(w)P_i^s(w)}{N_0 + I_s(w) + I_p(w) - h_i^{sb}(w)P_i^s(w)} \tag{13}$$

$\forall i \in N_s \wedge x_{iw} = 1 \wedge hop(i) = 1,$

where \wedge denotes logical AND operator. For other cases when $hop(i) \neq 1$, the SINR of the admitted SU i to another admitted SU q at channel w can be defined as follows.

$$\xi_{iq}^s(w) = \frac{h_{iq}^{ss}(w)P_i^s(w)}{N_0 + I_s(w) + I_p(w) - h_{iq}^{ss}(w)P_i^s(w)} \tag{14}$$

$\forall i \in N_s \wedge x_{iw} = 1 \wedge hop(i) \neq 1,$

where $h_{iq}^{ss}(w)$ denote the power attenuation on the channel w from SU i to SU q .

Similarly as Equation 11, $h_{iq}^{ss}(w)$ can be calculated as follows.

$$h_{iq}^{ss}(w) = \frac{G_i^s(w)G_q^s(w)}{(d_{iq}^{ss})^\alpha}, \forall i \in N_s \wedge q \in \mathcal{I}(i), \tag{15}$$

where d_{iq}^{ss} is the distance from SU i to SU q .

D. QoS Requirements

One of the QoS metrics we consider in this work is the data transmission rate (DTR) which was also used in [10], [11], [15], however these studies are constrained to the single-hop scenario.

According to Shannon' channel capacity formulation, the maximum DTR λ can be estimated by

$$\lambda = B \log_2(1 + \xi) \tag{16}$$

where B is the channel bandwidth and ξ is the SINR.

In order to guarantee the minimum DTR $\lambda_i^{min,s}$ required by the admitted SU i and the minimum DTR $\lambda_k^{min,p}$ required by PT k , it is equivalent to guarantee the minimum SINRs $\xi_i^{min,s}$ for SU i and $\xi_k^{min,p}$ for PT k according to Equation 16.

$$\xi_i^s(w) \geq \xi_i^{min,s} = 2^{\frac{\lambda_i^{min,s}}{B}} - 1, \forall i \in N_s \wedge (x_{iw} = 1) \tag{17}$$

and

$$\xi_k^p(w) \geq \xi_k^{\min,p} = 2^{\frac{\lambda_k^{\min,p}}{B}} - 1, \forall k \in N_p^t. \quad (18)$$

Another QoS metric considered in this work is the maximum hop distance from any admitted SUs to the BS. Note that some admitted SUs may not be able to communicate with the BS directly, but rely on other admitted SUs to forward their packets to the BS. This raises another challenge, that is, maintaining the network connectivity induced by all admitted SUs. Due to the path loss and mobility of SUs, the hop distance $hop(i)$ for any admitted SU i should be also upper bounded by a predefined system threshold \mathfrak{k} , where $1 \leq \mathfrak{k} \leq 6$ in practice.

$$hop(i) \leq \mathfrak{k}, \forall i \in N_s \wedge (\exists w : (x_{iw} = 1)). \quad (19)$$

Combining with QoS requirement on DTR at admitted SUs, it also indicates that a delay-bounded routing scheme will be also reserved.

III. THE PROBLEM FORMULATION

To simplify our presentation, we model the multi-hop infrastructure-based SS of the Cogcell as an undirected connected graph $G = (V, E)$, where $V = \{BS\} \cup \{SUs\}$ represents the set of nodes that consists of BS and SUs and E contains unordered pairs of distinct nodes, such that $(v, u) \in E$ if only if the transmission from node v can directly reach node u and vice versa (the reachability of transmissions is assumed to be a symmetric relation). In other words, u has to be within v 's transmission range. In this case, we say that the nodes v and u are neighbors in G . The problem we investigate in this paper is to select a subset of SUs from the multi-hop infrastructure-based SS (e.g., graph G) and assign proper channels to the admitted SUs in order to maximize the coverage of the BS in terms of the total number of SUs admitted by the operator of a CogCell. Meanwhile, a QoS routing path will be also generated during the selection of admitted SUs, e.g., the admitted SUs also induce a connected subgraph. Moreover, the interference constraints at PRs, the QoS requirements for both PTs and admitted SUs can be also guaranteed. Note also that a delay-bounded routing scheme will also be generated due to the bounded maximum hops from any admitted SU to BS and guaranteed DTR at each admitted SUs. We formulate such a joint admission control, channel assignment, and QoS-aware routing scenario as a non-linear NP-hard problem as follows.

$$arg x_{iw}, y_{iq} \max \sum_{i=1}^{n_s} \sum_{w=1}^{n_w} x_{iw} \quad (20)$$

subject to:

$$\sum_{i=1}^{n_s} \sum_{w=1}^{n_w} \tau_{ij}^w + \sum_{k=1}^{n_p^t} \sum_{w=1}^{n_w} \zeta_{kj}^w \leq \Gamma_j^w, \forall j \in N_p^r, \forall w \in N_w \quad (21)$$

$$\xi_k^p(w) \geq \xi_k^{\min,p}, \forall k \in N_p^t \quad (22)$$

$$\xi_i^s(w) \geq \xi_i^{\min,s}, \forall i \in N_s \wedge (x_{iw} = 1) \wedge (hop(i) = 1) \quad (23)$$

$$\xi_{iq}^s(w) \geq \xi_i^{\min,s}, \quad (24)$$

$$\forall i \in N_s \wedge (x_{iw} = 1) \wedge (hop(i) > 1) \wedge (y_{iq} = 1)$$

$$\sum_{q=1}^{n_s} y_{iq} = 1, \quad (25)$$

$$\forall q \in N_s \wedge (hop(i) > hop(q)) \wedge ((i, q) \in E)$$

$$\wedge \left(\sum_{w=1}^{n_w} x_{qw} > 0 \right) \wedge \left(\sum_{w=1}^{n_w} x_{iw} > 0 \right)$$

$$hop(i) \leq \mathfrak{k}, \forall i \in N_s \wedge (\exists w : (x_{iw} = 1)). \quad (26)$$

$$\sum_{i=1}^{n_s} \tau_{iq}^w x_{iw} + \sum_{k=1}^{n_p^t} \zeta_{kq}^w \leq \Gamma_q^w, \quad (27)$$

$$\forall q \in N_s \wedge i \in \mathcal{J}(q) \wedge k \in \mathcal{J}(q) \wedge (\exists i : y_{iq} = 1)$$

$$\sum_{w=1}^{n_w} x_{iw} \leq 1, \forall i \in N_s \quad (28)$$

$$x_{iw} \in \{0, 1\}, \forall i \in N_s, \forall w \in N_w \quad (29)$$

$$y_{iq} \in \{0, 1\}, \forall i \in N_s, \forall q \in N_s \quad (30)$$

$$\xi_i^s(w) = 0, \forall i \in N_s \wedge (x_{iw} = 0) \quad (31)$$

$$P_i^s(w) = 0, \forall i \in N_s \wedge (x_{iw} = 0) \quad (32)$$

where the first constraint (Equation 21) says that the interference to any PR j due to the transmissions from all admitted SUs and PTs sharing the same channel with PR j can not exceed the predefined system interference threshold. The second constraint (22) mentions that the QoS requirements at PTs in terms of the minimum DTR should be guaranteed. The following two constraints (Equations 23 and 24) remark that the QoS at admitted SUs in terms of the minimum DTR at the links on the selected routing paths should be satisfied, e.g., the connectivity of the subsystem induced by admitted SUs will be guaranteed. The binary variable x_{iw} is used to indicate whether SU i is admitted to access the channel w to the system, e.g., $x_{iw} = 1$ means that SU i is admitted on channel w , and $x_{iw} = 0$ indicates that SU i is not allowed to use channel w in the CogCell system. The binary variable y_{iq} is used to indicate whether the link $(i, q) \in E$ is selected by the routing algorithm. The constraint in Equation 25 provides out routing strategy, e.g., assuming a single path routing. Note that the multi-path or load-balanced routing schemes are not our focus in this work however, it is not difficult to built existing routing strategies into our framework. The constraint in Equation 26 gives an upper bound on the hop distance $hop(i)$ for any admitted SU i to BS, e.g., upper bounded by a predefined system threshold \mathfrak{k} . The constraint in Equation 27 says that the interference power accumulated at any admitted SU q on the selected routing path due to the transmissions from all admitted SUs and PTs in its interference range using the same channel should not exceed the predefined system interference threshold. The constraint in Equation 28 ensures that only one channel can be used by any admitted SUs. The following two constraints (Equations 29 and 30) define the values of the

variables x_{iw} and y_{iq} . The last two constraints (Equation 31 and 32) clarify the situation when SU i is forbidden to access any channels in the system.

Theorem 1: The coverage optimization problem we defined above is NP-hard.

Proof: It directly follows since a special case (e.g., single-hop) of the scenario we considered here had been proved to be a NP-hard problem in the literature. ■

IV. JOINT ADMISSION CONTROL, CHANNEL ASSIGNMENT, AND QoS ROUTING

In this section, we present three new joint admission control, channel assignment, and QoS routing algorithms which includes a greedy heuristic $O(\frac{1}{(n_p^r+n_s)n_w})$ approximation scheme (GHAS-M), a $O(\frac{1}{(\log(n_p^r+n_s)+\log n_w)^2})$ approximation algorithm (Fast-AA-M), and an exact solution for multi-hop CogCell systems.

A. A Greedy Heuristic Algorithm

In this section, we propose a greedy heuristic approximation algorithm (GHAA-M) to solve the coverage optimization problem by joint consideration of admission control, channel assignment, and QoS routing strategy in multi-hop CogCells. To simplify our presentation, we also define some new notions that will be used in our approximation schemes later. A dormant SU node is an SU that has already been admitted into the CogCell system, and an active SU node is an SU which is not yet dormant. Let D^s , A^s , and $num(i)$ denote the set of dormant nodes in the SS, the set of active nodes, and the number of active SUs with the hop distance at most \mathfrak{k} to the BS through the SU $i \in A^s$ respectively. For the sake of technique, we also assume that the BS of the CogCell is a dormant node in the SS. In our GHAA-M scheme, the selection of admitted SUs $i \in N_s$, assignment of the channels w for the admitted SUs, and the selection of links on the routing paths from an admitted SU i to the BS in a multi-hop manner are crucially based on a preference function $e_{iq}(w)$ that is described as follows, where $i \in A^s$ and $q \in D^s$, e.g., initially only $BS \in D^s$.

$$e_{iq}(w) = \frac{num(i)}{E_{ij}^p(w)E_{iq}^s(w)},$$

$$\forall i \in A^s, \forall q \in D^s \wedge (i, q) \in E, \forall w \in N_w, \forall j \in N_p^r(w),$$
(33)

where

$$E_{ij}^p = \sum_{j \in N_p^r(w)} (\sum_{i \in A^s} \tau_{ij}^w - (\Gamma_j^w - \sum_{k \in N_p^t(w)} \zeta_{kj}^w)) \tau_{ij}^w, \quad (34)$$

$$E_{iq}^s(w) = \sum_{q \in D^s} (\sum_{i \in A^s} \tau_{iq}^w - (\Gamma_q^w - \sum_{k \in N_p^t(w)} \zeta_{kq}^w)) \tau_{iq}^w. \quad (35)$$

From the preference function $e_{iq}(w)$ 33, it is easy to see that an active SU who has more active \mathfrak{k} -hop neighbors or less signal and interference power to PTs and the admitted SUs has more chance to be admitted to the system. Having $e_{iq}(w)$, the selection of SUs to be admitted to the CogCell system can

be done in a greedy manner. In each iteration of GHAA-M, an unique SU i with the largest value of $e_{iq}(w)$ will be admitted to access the channel w in the CogCell if the acceptance of SU i can still guarantee the QoS constraints at PTs, PRs, and the admitted SUs $q \in D^s$, where $w \in N_w$. Meanwhile, the selection approach with the largest $e_{iq}(w)$ under guaranteed QoS also generates a routing scheme, e.g., the link $(i, q) \in E$ will be selected to forward the packages for SU i to the BS. The details of GHAA-M are illustrated in Algorithm 1.

Algorithm 1 Greedy Heuristic Approximation Algorithm for Coverage Optimization in Multi-hop CogCells

Input: $N_w, N_p^t(w), N_p^r(w), N_s, G(V, E), \Gamma_j^w, h_i^{sb}(w), h_{ij}^{sp}(w), h_{kj}^{pp}(w), h_k^{pb}(w), d_i^{sb}, d_{ij}^{sp}, d_{iq}^{ss}, d_{kj}^{pp}, d_k^{pb}, d_{kq}^{ps}, d_{iq}^{ss}, \alpha, G_i^s(w), G_k^{pt}(w), G_j^{pr}(w), \mathfrak{k}, P_k^p(w), P_i^s(w)$.

Output: $D^s, x_{iw}(i \in D^s \wedge w \in N_w), y_{iq}(i \in D^s \wedge q \in D^s)$, and Max_s (the number of admitted SUs).

- 1: $Max_s = 0; D^s = \{BS\}; hop(BS) = 0;$
 - 2: **for** $i = 1$ to n_s **do**
 - 3: **for** $w = 1$ to $|N_w|$ **do**
 - 4: $x_{iw} = 0;$
 - 5: **for** $q = 1$ to n_s **do**
 - 6: $y_{iq} = 0;$
 - 7: **repeat**
 - 8: **for** $w = 1$ to $|N_w|$ **do**
 - 9: **for each** $q \in D^s \wedge hop(q) < \mathfrak{k}$ **do**
 - 10: **for each** $i \in N_s \wedge i \notin D^s$ **do**
 - 11: **if** $(i, q) \in E$ **then**
 - 12: compute $e_{iq}(w)$ according to Equation 33;
 - 13: Select a SU i with largest value of $e_{iq}(w)$ and the QoS constraints at PTs (Equation 22), PRs (Equation 21) and admitted SUs (Equations 23 and 24) still hold due to the acceptance of i to the CogCell system;
 - 14: **if** $i \neq \emptyset$ **then**
 - 15: $hop(i) = hop(q) + 1;$
 - 16: $D^s = D^s + \{i\};$
 - 17: $x_{iw} = 1;$
 - 18: $y_{iq} = 1;$
 - 19: **until** $(\forall q \in D^s : hop(q) \geq \mathfrak{k})$ or $(i = \emptyset)$
 - 20: **for each** $i \in N_s \wedge i \notin D^s$ **do**
 - 21: **for** $w = 1$ to $|N_w|$ **do**
 - 22: $P_i^s(w) = 0;$
 - 23: $Max_s = \sum_{i=1}^{n_s} \sum_{w=1}^{n_w} x_{iw};$
-

The GHAA-M scheme we propose here is very simple and easy to be implemented however it is not difficult to see that GHAA-M in the worst case scenario can lead a $O(\frac{1}{(n_p^r+n_s)n_w})$ -approximation ratio if all admitted SUs have the largest interference component on selected channel w at a particular PR $j \in N_p^r(w)$ or an admitted SU $q \in D^s \wedge \sum_{i=1}^{n_s} y_{iq} > 0$. Motivated by the disadvantage of GHAA-M, we also propose another approximation algorithm with a much better approximation ratio guarantee in the following Section IV-B.

B. A $O(\frac{1}{(\log(n_p^r+n_s)+\log n_w)^2})$ -approximation algorithm

In this section, we present a new polynomial-time joint admission control, channel assignment, and QoS routing scheme called Fast-AA-M for multi-hop CogCells in which our algorithm can guarantee that the number of SUs admitted to the CogCell system by the operator is at least $O(\frac{1}{(\log(n_p^r+n_s)+\log n_w)^2})$ fraction of any optimum solution meanwhile the QoS requirements from PTs, PRs and all admitted SUs are also guaranteed. Our new coverage optimization algorithm is partially based on the frame work for set covering problems which is called ρ -approximation subset oblivious algorithm in [3], and a new observation that $n_w n_p^r$ -dimension (multiple multi-dimension) bin packing is a $O(n_w n_p^r)$ -approximation subset oblivious in [10]. However, the work in [10] investigated only for single-hop scenario and the proposed approach in [10] can not be directly implied to multi-hop scheme with a guaranteed performance ratio. The details of our Fast-AA-M will be given in Section IV-B2. Prior to presenting our new scheme, we will introduce some terminologies and approaches in Section IV-B1 that will be used as a sub-procedure in our Fast-AA-M later.

1) *Preliminaries: For the completeness of our presentation, we re-produce the ρ -approximation subset oblivious algorithm from [3] and a new observation from [10] that will be used as a sub-procedure in our Fast-AA-M scheme in Section IV-B2.*

Given a set I of d -dimensional items, the i -th corresponding to a d -tuple $(t_i^1, t_i^2, \dots, t_i^d)$, that must be packed into the smallest number of unit-size bins, corresponding to the d -tuple $(1, \dots, 1)$. Given an instant I , let $opt(I)$ denote the value of the optimal solution for I . This problem can be formulated as the following general set covering problem, in which a set I of items has to be covered by configurations from the collection $\mathcal{C} \subseteq 2^I$, where each configuration $C \in \mathcal{C}$ corresponds to a set of items that can be packed into a bin:

$$\min\left\{\sum_{C \in \mathcal{C}} y_C : \sum_{i \in C} y_C \geq 1 (i \in I), y_C \in \{0, 1\} (C \in \mathcal{C})\right\} \quad (36)$$

Since the collection \mathcal{C} is exponentially large for the given application item set I , an approximation algorithm (or LP relaxation of 36) can be very useful for such an application.

The dual of this LP (Equation 36) is given by

$$\max\left\{\sum_{i \in I} w_i : \sum_{i \in C} w_i \leq 1 (C \in \mathcal{C}), w_i \geq 0 (i \in I)\right\} \quad (37)$$

Note that the separation problem for the dual is the following knapsack-type problem: given weights w_i on each item i , find a feasible configuration in which the total weight of items does not exceed 1. In the literature, it has been shown that:

Theorem 2: If there exists a Polynomial-Time Approximation Scheme (PTAS) for the separation problem for 37, that is given $w_i \in \mathbb{R}_+^{|I|}$ solve $\max_{C \in \mathcal{C}} \sum_{i \in C} w_i$, then there exists a PTAS for the LP relaxation of 36.

Based on Theorem 2, an approximation solution of the set covering problem 36 has been constructed in [3], which

consists of the following steps, where $\delta > 0$ is a parameter whose value can be specified later.

Step 1: Solve the LP relaxation of 36, possibly approximately in case \mathcal{C} is exponentially large in the input size. Let y^* be the (near-)optimal solution of the LP relaxation and $z^* = \sum_{C \in \mathcal{C}} y_C^*$ be its value. Let $C_1, C_2, \dots, C_m \in \mathcal{C}$ be the configurations associated with the nonzero components of y^* ;

Step 2: Define the binary vector y^r starting with $y_C^* = 0$ for $C \in \mathcal{C}$ and $S = I$ (i.e., all items are uncovered) and then repeat the following for $\lceil \delta z^* / (1 - \sigma\psi/2) \rceil$ iterations: select the configuration $C' \in \{C_1, C_2, \dots, C_m\}$ such that $\Phi(S - C')$ is minimum and let $y_{C'}^r = 1$ and $S = S - C'$, where σ is a small parameter such that $\sigma\psi < 1$ to be specified later. For an arbitrary set of items S , $\Phi(S) = \ln(\sum_{j=1}^d e^{\sigma \sum_{i \in S} w_i^j})$;

Step 3: Consider the set of items $S \subseteq I$ that are not covered by y_r , namely $i \in S$ if and only if $\sum_{i \in C} y_C^r = 0$, and the associated optimization problem for the residual instance

$$\min\left\{\sum_{C \in \mathcal{C}} y_C : \sum_{i \in C} y_C \geq 1 (i \in S), y_C \in \{0, 1\} (C \in \mathcal{C})\right\}. \quad (38)$$

Apply some approximation algorithm to the problem 38 yielding solution y^a ;

Step 4: Return the solution $y^h = y^r + y^a$.

For the abbreviation, we denote this approach by SETCOVER(I, w), where w is the weight vector for $\forall i \in I$.

A crucial notation used in [3] is called the ρ -approximation subset oblivious algorithm which is defined as follows.

Definition 3: A ρ -approximation algorithm for problem 36 is called subset oblivious if, for any fixed $\varepsilon > 0$, there exist constraints d, ψ, ϖ (possibly depending on ε) such that, for every instance I of 36, there exist vectors $w^1, w^2, \dots, w^d \in \mathbb{R}^{|I|}$ with the following properties: (i) $\sum_{i \in C} w_i^d \leq \psi$, for each $C \in \mathcal{C}$ and $j = 1, 2, \dots, d$; (ii) $opt(I) \geq \max_{j=1}^d \sum_{i \in I} w_i^j$; (iii) $appr(S) \leq \rho \max_{j=1}^d \sum_{i \in S} w_i^j + \varepsilon opt(I) + \varpi$, for each $S \subseteq I$.

The following theorem has been shown in [3].

Theorem 4: A ρ -approximation subset oblivious problem can be solved by procedure SETCOVER(I, w) with $O(\log \rho)$ approximation ratio guarantee.

In what follows, we introduce a new observation from [10] (e.g., an extension from [3]) which will be employed as a sub-procedure in our new Fast-AA-M scheme to guarantee a proper performance in the worst case scenario.

Theorem 5: ι d -dimensional (multiple multi-dimensional) bin packing is a $O(\iota \cdot d)$ -approximation subset oblivious algorithm.

Theorem 6: The cost of the final heuristic solution for ι d -dimensional bin packing produced by procedure SETCOVER(I, W) with $\delta = \ln d \cdot \iota, \sigma = (2\varepsilon / \ln d \cdot \iota) / (\psi + \psi\varepsilon / \ln d \cdot \iota)$ and $\psi = 1$ is at most

$$(\ln d \cdot \iota + 1 + 2\varepsilon)opt(I) + \delta + \frac{(2 \ln d \cdot \iota)(1 + \varepsilon / \ln d)}{(2\varepsilon / \ln d \cdot \iota)} + 1, \quad (39)$$

i.e., this is a deterministic $O(\log d + \log \iota)$ -approximation algorithm for ι d -dimensional bin packing (e.g., problem 36).

2) *The Fast-AA-M Scheme*: In this section, we present our main approximation algorithm (Fast-AA-M) with $O(\frac{1}{(\log(n_p^r+n_s)+\log n_w)^2})$ approximation ratio guarantee.

The main idea of our algorithm is that we first select a subset of SUs out of all active SUs to form a connected backbone network $G_s^{bone} = (V_s^{bone}, E_s^{bone})$ in which the sub-graph induced by G_s^{bone} and its immediate neighbors of active SUs can cover at least a logarithmical fraction of the total number of admitted SUs by any optimal solution for the coverage optimization problem in the worst case scenario, where $V_s^{bone} \subseteq V$ and $E_s^{bone} \subseteq E$. The construction of G_s^{bone} is based on a smart use of the multiple multi-dimensional bin packing approach in [10]. Moreover, we also ensure that acceptance of all SUs in the backbone network G_s^{bone} will not cause any trouble to guarantee the QoS requirements at PUs during the construction of G_s^{bone} . Although the existing minimum connected dominating set approximation schemes can be used to construct the G_s^{bone} , the QoS requirements at PUs and admitted SUs may not be guaranteed, e.g., the nodes in the connected dominating set can not be all admitted to the CogCell system. Based on this constructed backbone network, we convert a variety of our problem to a n_w ($n_p^r + n_s^{bone}$)-dimensional (multiple multi-dimensional) bin packing problem in which each potentially admitted SUs with one-hop connection to G_s^{bone} corresponds to the items and all PRs, the SUs in G_s^{bone} , and the available channel n_w corresponds to the n_w ($n_p^r + n_s^{bone}$)-dimensional bins, where n_s^{bone} is the number of admitted SUs in G_s^{bone} . After that, one more execution of the approach in [10] based on the constructed backbone network will derive the expected solution. The details of our Fast-AA-M are illustrated in Algorithm 2.

According to the Equations 8 and 10, we can derive an upper bound of signal and interference power allowed by all admitted SUs at each PT k , e.g., the QoS at PT k can not be guaranteed if the signal and interference power from all admitted SUs exceeds this bound.

$$I_s^k(w) = \frac{h_k^{pb}(w)P_k^p(w)}{\xi_k^p(w)} - N_0 - \sum_{\substack{1 \leq u \leq n_p^t \\ u \neq k}} h_u^{pb}(w)P_u^p(w) - \sum_{\forall q \in V_s^{bone}} h_{qk}^{sp}(w)P_q^s(w), \forall k \in N_p^t(w). \quad (40)$$

Another upper bound of signal and interference power of all admitted SUs at each PR $j \in N_p^r(w)$ can be achieved as follows according to Equation 5.

$$I_s^j(w) \leq \Gamma_j^w - \sum_{k=1}^{n_p^t(w)} \zeta_{kj}^w - \sum_{\forall q \in V_s^{bone}} \tau_{qj}^w \\ = \Gamma_j^w - \sum_{k=1}^{n_p^t(w)} \frac{G_k^{pt}(w)G_j^{pr}(w)P_k^p(w)}{(d_{kj}^{pp})^\alpha} - \sum_{\forall q \in V_s^{bone}} \tau_{qj}^w, \forall j \in N_p^r(w). \quad (41)$$

Similarly, the upper bound of signal and interference power caused by admitted SUs at each admitted SU q in the backbone network G_s^{bone} can be bounded by:

$$I_s^q(w) \leq \Gamma_q^w - \sum_{k=1}^{n_p^t(w)} \zeta_{kq}^w - \sum_{\substack{\forall u \in V_s^{bone} \\ u \neq q}} \tau_{uq}^w, \forall q \in V_s^{bone}. \quad (42)$$

Note that a potentially admitted SU i refers to an SU that satisfies the following constraints (Equation 43):

$$\forall i : i \in N_s \wedge (\tau_{ij}^w \leq I_s^j(w)) \wedge (\tau_{iq}^w \leq I_s^q(w)) \\ \wedge (\tau_{ik}^w \leq I_s^k(w)) \wedge (\exists q \in V_s^{bone} : (i, q) \in E), \quad (43) \\ \forall j \in N_p^r(w), \forall q \in V_s^{bone}, \forall k \in N_p^t(w), \exists w \in N_w.$$

Consequently, all PRs and admitted SUs $q \in V_s^{bone}$ on particular channel w corresponds to a $(n_p^r + n_s^{bone})$ -tuple unit-size bin $(1, \dots, 1)$, and a potential admitted SU i corresponds to a $(n_p^r + n_s^{bone})$ -tuple item $(W_i^1, W_i^2, \dots, W_i^{n_p^r}, W_i^{(n_p^r+1)}, \dots, W_i^{(n_p^r+n_s^{bone})})$, where

$$W_i^z(w) = \frac{\tau_{iz}^w}{\Gamma_z^w - \sum_{k=1}^{n_p^t(w)} \zeta_{kj}^w - \sum_{\forall q \in V_s^{bone}} \tau_{qj}^w}, 1 \leq z \leq n_p^r, \quad (44)$$

and

$$W_i^z(w) = \frac{\tau_{iz}^w}{\Gamma_z^w - \sum_{k=1}^{n_p^t(w)} \zeta_{kz}^w - \sum_{\substack{\forall u \in V_s^{bone} \\ u \neq q}} \tau_{uq}^w}, \quad (45) \\ \forall z : (n_p^r + 1) \leq z \leq (n_p^r + n_s^{bone}).$$

Note that $W_i^j(w)$ can be different for different channels. After execution of the heuristic algorithm for n_w ($n_p^r + n_s^{bone}$)-dimensional bin packing produced by procedure SETCOVER($(N_s - V_s^{bone}), w$), combining with QoS requirements at each admitted SU i , we select one bin for each channel w which can achieve the maximal number of admitted SUs among all bins configured for each channel $w \in N_w$.

Due to the construction of the admitted SUs, we know that there always exists a feasible routing paths from any admitted SU to the BS meanwhile the DTR at each link on this path and length of such a path in terms of number of hops will satisfy the QoS requirements we defined at Section III. Consequently, Dijkstra's algorithm can be used to figure out the exact path for each admitted SUs to the BS.

Theorem 7: Our Fast-AA-M scheme can guarantee that the number of admitted SUs achieved is at least $O(\frac{1}{(\log(n_p^r+n_s)+\log n_w)^2})$ fraction of the one of any optimal solution meanwhile the QoS required by all PTs, PRs and the admitted SUs are also satisfied.

Proof: The QoS requirements at all PTs, PRs and the admitted SUs can be guaranteed due to the construction of the set of admitted SUs, e.g., acceptance of an SU to the CogCell system if only if QoS requirements at PUs and all admitted SUs are satisfied. During the construction of backbone network in Algorithm 2, we use the multiple multi-dimensional bin packing approach from [10] and we select

Algorithm 2 Fast Approximation Algorithm (Fast-AA-M) for Multi-hop CogCells

Input: Same as the input in Algorithm 1.

Output: D^s , x_{iw} ($i \in D^s \wedge w \in N_w$), y_{iq} ($i \in D^s \wedge q \in D^s$), and Max_s (the number of admitted SUs).

```

1:  $D^s = V_s^{bone} = \{BS\}; hop(BS) = 0; flag = 0;$ 
2: for  $i = 1$  to  $n_s$  do
3:   for  $w = 1$  to  $|N_w|$  do
4:      $x_{iw} = 0;$ 
5:   for  $q = 1$  to  $n_s$  do
6:      $y_{iq} = 0;$ 
7: repeat
8:    $N_s^{temp} = \{i : (i \in N_s) \wedge (\exists q : q \in D^s \wedge (hop(q) <$ 
    $\mathfrak{k})) \wedge (i, q) \in E\};$ 
9:   for  $w = 1$  to  $|N_w|$  do
10:    for each  $i \in N_s^{temp}$  do
11:      compute  $W_i^j(w)$  according to 44 and 45;
12:    execute SETCOVER( $N_s^{temp}, W$ );
13:    if SETCOVER( $N_s^{temp}, W$ )  $\neq \emptyset$  then
14:      for  $w = 1$  to  $|N_w|$  do
15:        select one bin  $\mathfrak{B}$  with maximal  $\sum_{vi \in \mathfrak{B}} num(i)$  on
        channel  $w$ ;
16:        for  $i \in \mathfrak{B}^{(w)}$  do
17:          if (the QoS at PUs and admitted SUs  $q \in D^s$ 
          can still be guaranteed with acceptance of SU
           $i) \wedge (\exists q : q \in D^s \wedge \xi_{iq}^s(w) \geq \xi_i^{min,s})$  then
18:            select a SU  $q$  with  $Max_{q \in D^s} \xi_{iq}^s(w) \wedge$ 
            ( $hop(q) < \mathfrak{k}$ );
19:             $hop(i) = hop(q) + 1;$ 
20:             $D^s = V_s^{bone} = D^s + \{i\};$ 
21:             $x_{iw} = 1;$ 
22:             $y_{iq} = 1; E_s^{bone} = E_s^{bone} + \{(i, q)\};$ 
23:          until ( $\forall q : (q \in D^s) \wedge (hop(q) \geq \mathfrak{k})$ ) or
          (SETCOVER( $N_s^{temp}, W$ ) =  $\emptyset$ )
24:          if  $flag = 1$  then
25:            go to 34;
26:          for each  $q \in V_s^{bone}$  do
27:             $count(q) = \sum_{vi \in V_s^{bone}} (y_{qi} + y_{iq});$ 
28:          for each  $i \in V_s^{bone}$  do
29:            if count(i)=1 then
30:               $V_s^{bone} = V_s^{bone} - \{i\};$ 
31:               $E_s^{bone} = E_s^{bone} - \{(i, q)\}, \forall (i, q) \in E_s^{bone}; y_{iq} = 0;$ 
32:             $D^s = V_s^{bone}; flag = 1;$ 
33:          go to 7 :
34:          for each  $i \in N_s \wedge i \notin D^s$  do
35:            for  $w = 1$  to  $|N_w|$  do
36:               $P_i^s(w) = 0;$ 
37:             $Max_s = \sum_{i=1}^{n_s} \sum_{w=1}^{n_w} x_{iw};$ 

```

one bin with maximal number of SUs to be admitted to the system in each iteration of the construction that can guarantee that the coverage of the backbone network in terms of the number of admitted SUs within one-hop connection to the backbone network and the number of admitted SUs in the

backbone network is at least $\Omega(\frac{1}{\log(n_p^r + n_s) + \log n_w})$ fraction of the optimum due to Theorem 6. According to Theorems 4, 5 and 6, the procedure SETCOVER($(N_s - V_s^{bone}), w$) will further lead a logarithmical factor down in terms of the number of admitted SUs. Combining with the polynomial-time complexity of the approach in [10], It completes the proof. ■

C. An Exact Solution

Due to the enormous memory requirement by the dynamic programming approaches to achieve an optimal solution for the problem formulated in Section III, we employ the standard branch-and-bound approach in [6] to solve our problem. As mentioned in [6], [10], the crucial property of branch-and-bound techniques is an intelligent enumeration of the solution space for the optimization problems. It can divide the original problem into several decomposed subproblems and calculate these subproblems in parallel which significantly reduce the computational burden. Based on the standard methodologies introduced in [6] and a framework for single-hop scenario in [10], the original optimization problem investigated here can be transferred to the following problem with exactly the same constraints as the original ones.

$$arg_{x_{iw}, y_{iq}} max(\mathbb{L}(\mathfrak{X}, \mathfrak{Y}, \mathfrak{r}, \mathfrak{\eta})) \quad (46)$$

where

$$\begin{aligned} \mathbb{L}(\mathfrak{X}, \mathfrak{Y}, \mathfrak{r}, \mathfrak{\eta}) = & \sum_{i=1}^{n_s} (\sum_{w=1}^{n_w} x_{iw} + \sum_{q=1}^{n_s} y_{iq}) \\ & - \sum_{i=1}^{n_s} \mathfrak{r}_i (\sum_{w=1}^{n_w} x_{iw} + \sum_{q=1}^{n_s} y_{iq} - 2) - \\ & \sum_{w=1}^{n_w} \mathfrak{\eta}_w \sum_{i=1}^{n_s} (\tau_{ij}^w + \sum_{k=1}^{n_p^t} \zeta_{kj}^w + \sum_{q=1}^{n_s} \tau_{qj}^w - \Gamma_j^w), \forall j \in N_p^r, \end{aligned} \quad (47)$$

where \mathfrak{r}_i and $\mathfrak{\eta}_w$ are dual variables. It is easy to see that the optimal solution of the dual problem (Eq. 46) is an upper bound of the optimum of the original problem for arbitrary nonnegative \mathfrak{r}_i and $\mathfrak{\eta}_w$. However, to achieve an optimal solution in terms of a tight upper bound, the optimum dual variables have to be chose such that $\mathbb{L}(\mathfrak{X}, \mathfrak{Y}, \mathfrak{r}, \mathfrak{\eta})$ (Eq. 47) is minimized. The details of standard branch-and-bound approach can be found in [6].

V. CONCLUSION

In this paper, we mathematically formulate the problem of joint admission control, channel assignment and QoS routing to maximize the coverage of BS for SUs in the CogCell system that supports multi-hop secondary transmissions, taking into account the interference constraints and QoS requirements from both PUs and admitted SUs. To our best knowledge, we have made the first effort in studying this important problem. It is worth mentioning that none of the work in the literature so far concurrently investigate the issues of the three procedures in multi-hop infrastructure-based secondary systems. Particularly, multi-hop routing related issues are rarely studied. In this paper, We propose three different algorithms to solve

the coverage optimization problem and analysis its theoretical performance in terms of approximation ratio to the optimum which include a greedy heuristic scheme GHAA-M, an algorithm providing an exact solution, and more importantly an approximation solution Fast-AA-M with a poly-logarithmic approximation ratio guarantee. Our preliminary simulation results also indicate that all new approximation algorithms we developed in this work can effectively exploit the increased number of SUs and channels, and perform much better indeed than the theoretical worst case bound.

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TABLE I
TERMINOLOGIES AND NOTATIONS

Term	Definition
CogCell	cognitive radio cellular networks
SS	secondary system
QoS	quality of service
B	channel bandwidth
BS	base station
PU	primary user
SU	secondary user
PTs	PTs which are in transmitting mode
PRs	PTs which are in receiving mode
N_s	the set of SUs
N_p^t	the set of PTs
$N_p^r(w)$	the set of PTs on channel w
$N_p^t(w)$	the set of PRs on channel w
N_w	the set of the available channels
n_s	the number of SUs
n_p	the number of PTs
$n_p^t(w)$	the number of PTs on channel w
$n_p^r(w)$	the number of PRs on channel w
n_w	the number of available channels
$P_i^s(w)$	the transmission power of SU i on channel w
$P_k^p(w)$	the transmission power of PU k on channel w
$R_T(\mu)$	the transmission range of user μ
$R_I(\mu)$	the interference range of user μ
γ_μ	Interference-Transmission ratio for user μ
$\mathcal{S}(\mu)$	the subsets of users within the transmission range of μ
$\mathcal{I}(\mu)$	the subsets of users within the interference range of μ
τ_{ij}^w	the interference to PR j by SU i on channel w
$\zeta_{k,j}^w$	the interference to PR j by PT k on channel w
Γ_w^j	the interference threshold at PR j on channel w
$h_i^{sb}(w)$	the power attenuation from SU i to BS
$h_i^{sq}(w)$	the power attenuation from SU i to SU q
$h_{ij}^{sp}(w)$	the power attenuation from SU i to PR j
$h_{kj}^{pp}(w)$	the power attenuation from PT k to PR j
$h_k^{sb}(w)$	the power attenuation from PT k to BS
d_i^{sb}	the distance from SU i to BS
d_{iq}^{ss}	the distance from SU i to SU q
d_{ij}^{sp}	the distance from SU i to PR j
d_{kj}^{pp}	the distance from PT k to PR j
d_k^{sb}	the distance from PT k to BS
$I_s(w)$	the interference received at BS from all permitted SUs
$I_p(w)$	the interference received at BS caused by the PTs
SINR	signal-to-interference-plus-noise ratio [8]
$\xi_i^s(w)$	the SINR of SU i measured at BS on channel w
$\xi_k^p(w)$	the SINR of PT k measured at BS on channel w
$\xi_i^{min,s}$	the minimum SINR required by SU i
$\xi_k^{min,p}$	the minimum SINR required by PT k
$\lambda_i^s(w)$	the DTR achieved by SU i
$\lambda_k^p(w)$	the DTR achieved by PT k
$\lambda_i^{min,s}$	the minimum DTR required by SU i
$\lambda_k^{min,p}$	the minimum DTR required by PT k
$G_i^s(w)$	the antenna gain of SU i
N_0	background noise
$G_k^{pt}(w)$	the antenna gain of PT k
$G_j^{pr}(w)$	the antenna gain of PR j
α	the path fading factor
$T_j^I(w)$	the interference power accumulated at PR j on channel w
DTR	data transmission rate
hop(i)	hop distance from SU i to the BS
\dagger	predefined system threshold for the maximum hop bound
x_{iw}	binary variable indicates whether SU i can access channel w
y_{iq}	binary variable indicates whether link $(i, q) \in E$ is selected