Optimal Divisible Load Scheduling and Markov Chain Models

Mequanint A. Moges and
Thomas G. Robertazzi[1]
Department of Electrical and
Computer Engr.
Stony Brook University
Stony Brook, New York 11794
e-mail:
mmoges,tom@ece.sunysb.edu

Abstract —

In this short paper we consider the equivalence between various load-scheduling policies and continuous time Markov chains. It provides a basic unification of both divisible load scheduling and Markov chains for the first time in 15 years of research. This unification is suggested by the fact that both divisible load scheduling theory and Markovian queueing theory have common features.

I. Background and Results

Divisible load theory involves the optimal distribution of arbitrarily partitionable loads among a number of processors and links [1]. Past studies have showed that there are a variety of potential load distribution policies that minimize the processing finishing time and maximize speedup [2] but only using Gantt chart-like timing diagrams.

In this paper we introduce continuous time Markov models for various network topologies used in parallel and distributed systems. Since the original work of Erlang, Jackson in 1957, and later Gordon and Newel in 1967[3], researchers were able to produce elegant and tractable analytic solutions of the product form type using local linear balance equations. That is, for this product form class of queueing networks any state equilibrium probability is a product of system parameters and a reference probability. With this in mind, to illustrate the equivalence we began our study with a basic tree network as follows.

Consider a single level tree network with (N+1) processors and (N) links. All the processors are assumed to be equipped with front-end processors. From our initial studies we have been able to show the equivalence between the recursive equations from the Gantt chart-like timing diagrams and the local balance equations of corresponding Markov chain models. Some resulting Markov models for N=3 are illustrated in Fig. 1. It is interesting to see from the results that the models have surprisingly simple flow structures, especially when homogeneous networks are considered.

The equivalence can also be readily extended to models without front-end processors as well as other network topologies. In our study we considered the linear network and multilevel tree network topologies with front-end processors. In the linear network case we considered the case of homogeneous communication link speed Z for all the processors. On the other hand for the case of multilevel tree networks, we have considered a two-level network with all parent processors having a maximum of two child processors. We have also

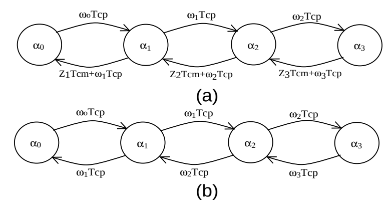

Figure 1: Markov chain model for single level tree - (a) sequential distribution of load over links (b)simultaneous distribution of load over links with processing starting as load received.

assumed that any processor in the network starts computing immediately after receiving its own share of load. This processor will be communicating with its child processors (if any) while doing its computation. Under these assumptions only, we have been able to find the equivalent Markov models for those network topologies.

II. Implication

The possible existence of relatively simple Markovian models may enable the development of computationally elegant and fast algorithms for performance evaluation through these new models. An immediate extension of this initial work would be to examine models for heterogeneous link speed linear networks, multi-level fat tree networks and others. Specific exciting open questions arise concerning the general relationship existing between processing topologies and their Markov models and as to whether it would be able to associate all possible load sharing policies to corresponding Markovian models or not.

References

[1] V. Bharadwaj, D. Ghose, T.G. Robertazzi "Divisible Load Theory: A New Paradigm for Load Scheduling in Distributed Systems," *Cluster Computing*, vol. 6, no. 1,Jan. 2003, pp. 7-18.

[2] V. Bharadwaj, D. Ghose, D. Mani, T.G. Robertazzi "Scheduling Divisible Loads in Parallel and Distributed Systems," *IEEE Computer Society Press*, Los Alamitos CA, 1996.

[3] W.J. Gordan and Newell G.F. Newell, "Closed Queueing Systems with Exponential Servers," *Operations Research*, vol. 15, 1967, pp. 254-265.

[1]The support of NSF grant CCR-99-12331 is acknowledged.