Problem 1

Let $f(E)$ be the Fermi distribution function, characterized by some (unspecified) Fermi energy E_F. Calculate the energy range (in eV) between $f(E) = 0.01$ and $f(E) = 0.99$

(a) for T = 300 K
(b) for T = 77 K

Solution.

From $f = [1 + e^{\beta(E - E_F)}]^{-1}$ we find

$$\Delta E \equiv E_{0.99} - E_{0.01} = kT\ (\ln 99 - \ln 1/99) = 2kT \ln(99)$$

(a) $T = 300\,\mathrm{K}$: $\Delta E = 0.238\,\mathrm{eV}$;
(b) $T = 77\,\mathrm{K}$: $\Delta E = 0.061\,\mathrm{eV}$

Problem 2

Determine the difference $E_C - E_F$ and the hole concentration p_0 from the known temperature T and electron concentration n_0 in silicon at equilibrium:

(a) for T = 300 K and $n_0 = 10^{15}\ \mathrm{cm}^{-3}$
(b) ☛ for T = 77 K and $n_0 = 10^{19}\ \mathrm{cm}^{-3}$

Solution.

(a) Nondegenerate statistics, $n_0 = N_C\, e^{\beta(E_C - E_F)}$, where $N_C = 2.8 \cdot 10^{19}\ \mathrm{cm}^{-3}$. Whence $E_C - E_F = 0.265\,\mathrm{eV}$. Next, $p_0 = n_i^2/n_0 = 2.25 \cdot 10^5\ \mathrm{cm}^{-3}$

(b) Degenerate statistics, use the Fermi integral. Since $N_C \propto T^{3/2}$, we have $N_C(77) = (77/300)^{3/2}\, N_C(300) = 3.6 \cdot 10^{18}\ \mathrm{cm}^{-3}$ and $n_0/N_C = 2.75$. This gives $E_F - E_C \approx 3.6\,\mathrm{kT} = 24\,\mathrm{meV}$. Next, from $n_i^2 \propto T^3\, e^{-E_G/kT}$ we find $n_i(77) = 5 \cdot 10^{-19}\ \mathrm{cm}^{-3}$ (neglecting the temperature variation of the bandgap itself). Whence $p_0 = n_i^2/n_0 = 2.45 \cdot 10^{-56}\ \mathrm{cm}^{-3} = 2.45 \cdot 10^{-41}/\mathrm{km}^3$, a very small number indeed.

Problem 3

A bar of silicon at room temperature with a cross-sectional area $A = 10^{-4}\ \mathrm{cm}^2$ is known to contain a (uniform) hole concentration of $p = 4.5 \cdot 10^{15}\ \mathrm{cm}^{-3}$. In an applied electric field $F = 1.5\,\mathrm{kV/cm}$ the measured current is 45 mA.

(a) determine the average drift velocity of holes;
(b) ☛ calculate the hole diffusion coefficient.

Solution.

(a) From the definition $I = A\, e\, n\, v_{drift}$ we find $v_{drift} = 6.25 \cdot 10^5\,\mathrm{cm/s}$.

(b) From $v_{drift} = \mu E$ we find the mobility μ, whence from Einstein's relation $D = \mu kT/e \approx 10.5\,\mathrm{cm^2/s}$.

Problem 4

The total junction capacitance of a one-sided silicon *pn* junction at 300 K is measured at $V_R = 50\,\mathrm{mV}$ and found to be 1.3 pF. The junction area is $A = 10^{-5}\,\mathrm{cm}^2$ and the built-in voltage $V_{bi} = 0.95\,\mathrm{V}$. Determine the impurity concentration

(a) on the low-doped side of the junction.

(b) ☞ on the higher-doped side of the junction.

Solution.

(a) $V_{bi} + V_R = 1.0\,\mathrm{V}$; denoting by N_L the impurity concentration on the low side of the junction, we have

$$\frac{C}{A} = \left[\frac{\varepsilon e N_L}{2(V_{bi} + V_R)} \right]^{1/2} ,$$

whence $N_L = 2 \cdot 10^{17}\,\mathrm{cm}^{-3}$.

(b) $N_H \approx 10^{19}\,\mathrm{cm}^{-3}$ is found from the relation $V_{bi} = (kT/e)\ln(N_H N_L / n_i^2)$.

Problem 5

Consider an ideal *pn* junction diode at $T = 300\,\mathrm{K}$, operating in the forward-bias regime. Calculate the change in voltage ΔV that will cause an increase in current by a factor of 10.

Solution.

From the diode equation $I_1/I_2 = \mathrm{e}^{e\,\Delta V/n\,kT}$, where n is the diode ideality factor. For an ideal *pn* junction diode at high enough forward voltage, one has $n = 1$ whence we find $\Delta V \approx 60\,\mathrm{mV}$.

Problem 6

The reverse-saturation currents of an ideal Schottky diode and an ideal *pn* junction diode are, respectively, $5 \cdot 10^{-8}\,\mathrm{A}$ and $10^{-12}\,\mathrm{A}$ at $T = 300\,\mathrm{K}$. The diodes are connected in a circuit and driven by a constant current of $0.5\,\mathrm{mA}$ (in the forward direction). Determine (*i*) the current in each diode and (*ii*) the voltage across each diode for

(a) parallel connection of the two diodes.

(b) series connection of the two diodes.

Solution.

(a) $V_{\mathrm{Sch}} = V_{pn} = 0.24\,\mathrm{V}$, $I_{\mathrm{Sch}} \approx 5 \cdot 10^{-4}\,\mathrm{A}$, $I_{\mathrm{pn}} \approx 1 \cdot 10^{-8}\,\mathrm{A}$,

(b) $I_{\mathrm{Sch}} = I_{pn} = 5 \cdot 10^{-4}\,\mathrm{A}$, $V_{\mathrm{Sch}} = 0.24\,\mathrm{V}$, $V_{\mathrm{pn}} = 0.52\,\mathrm{V}$.

Problem 7

A silicon *npn* bipolar transistor at $T = 300\,\mathrm{K}$ has a uniformly doped base region. The cutoff frequency is $f_T = 500\,\mathrm{MHz}$ and is known to be limited by the base transit time.

(a) Estimate the base width.

(b) ☛ Mobility of minority electrons in the base of this transistor is known to be twice higher at $T = 75\,\mathrm{K}$ than at $T = 300\,\mathrm{K}$. Estimate the f_T at $T = 75\,\mathrm{K}$.

Solution.

(a) $W_B = \sqrt{D\tau}$, where $\tau = 1/2\pi f_T = 3.2 \cdot 10^{-10}\,\mathrm{s}$. For $D = 35\,\mathrm{cm}^2/\mathrm{s}$, $W_B \approx 1.5\,\mu\mathrm{m}$.

(b) From Einstein's relation $eD = \mu kT$, we have

$$\frac{D(T_1)}{D(T_2)} = \frac{T_1}{T_2} \cdot \frac{\mu(T_1)}{\mu(T_2)} = \frac{1}{4} \cdot \frac{2}{1} = \frac{1}{2} ,$$

i.e., in this example $D(75) = 0.5 \times D(300)$ and hence $f_T(75) \approx 250\,\mathrm{MHz}$.

Problem 8

Assume that each atom is a hard sphere in contact with its nearest neighbors. What percentage of the total unit cell volume is occupied in

(a) fcc lattice,

(b) bcc lattice,

(c) diamond lattice ?

Problem 9

A free electron and a photon have the same energy. At what value of energy (in eV) will the wavelength of the photon be ten times that of the electron ?

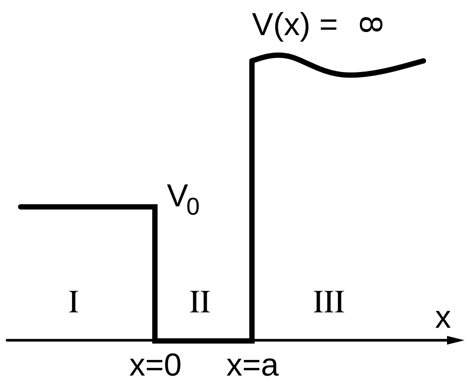
Problem 10

Consider the wave function $\psi(x,t) = A \sin(n\pi x)\, e^{-i\omega t}$ for $0 \le x \le 1$. Determine A so that the function is normalized:

$$\int_0^1 |\psi(x,t)|^2 \, dx = 1$$

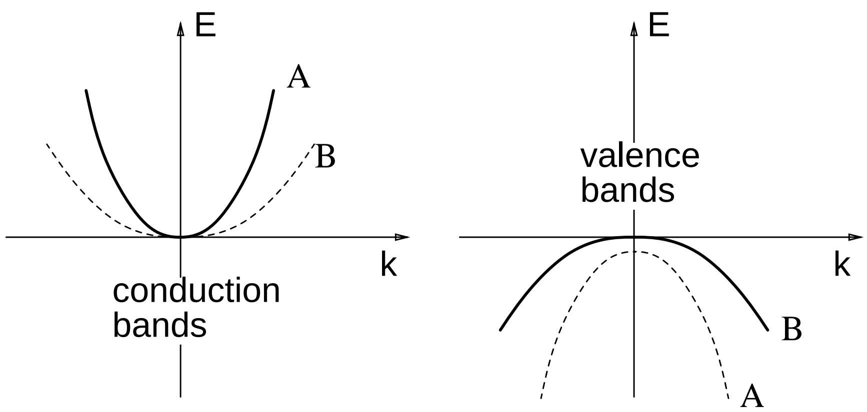
Problem 11

Consider the one-dimensional potential shown in the figure below. Assume the total electron energy is $E < V_0$.
(a) Write the wave function solutions which apply in each region.
(b) Write the set of equations which result from applying the boundary conditions.
(c) show explicitly why, or why not, the energy levels of the electron are quantized.



Problem 12

Two possible conduction bands (left figure) and valence bands (right figure) are shown in the E versus k diagrams below. State which bands will result in the heavier effective mass for electrons and holes, respectively. State why.



Problem 13

Show that the probability of an energy state ΔE above the Fermi energy being occupied is the same as the probability of a state ΔE below the Fermi energy being empty.

Problem 14

Calculate the temperature at which there is a 10^{-6} probability that an energy state $0.55\,\text{eV}$ above the fermi energy is occupied by an electron.

Problem 15

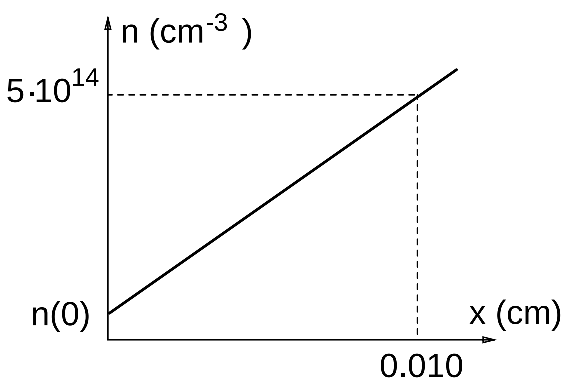
If the density of states function in a conduction band of a particular semiconductor is a constant equal to K, derive the expression for the thermal-equilibrium concentration of electrons in the conduction band,

(a) assuming Fermi-Dirac statistics;

(b) assuming Boltzmann approximation to be valid.

Problem 16

Consider a sample of silicon at $T = 300\,\text{K}$. Assume the electron concentration varies linearly with the distance, as shown in the Figure below. The electron diffusion current density equals $J = 0.19\,\text{A/cm}^2$. If the electron diffusion coefficient is $D = 25\,\text{cm}^2/\text{sec}$, find the electron concentration at $x = 0$.



Problem 17

The electron concentration in silicon at $T = 300\,\text{K}$ is given by

$$n(x) = 10^{16}\,e^{-x/18}\ \ [\text{cm}^{-3}]$$

where x is measured in μm and is limited to $0 \le x \le 25\,\mu$m. The electron diffusion coefficient is $D = 25\,\text{cm}^2/\text{sec}$. The total electron current density through the semiconductor is constant and equals $J = -40\,\text{A/cm}^2$. The electron current has both diffusion and drift components. Determine the electric field as a function of x that must exist in the semiconductor.