Problem 1 (Shur: 1-6-4)

Consider a free 2-dim electron gas confined to a quantum well of arbitrary shape and thickness. The eigenstates are labeled by the 2D wavenumber **k**, the subband index n and the spin s:

$$\Psi_{n,\,s,\,\mathbf{k}}\,(\mathbf{R}) = \chi_s\,\phi_n\,(z)\;\mathrm{e}^{\mathrm{i}\,\mathbf{k}\cdot\mathbf{r}}\;, \tag{1.1}$$

where $\mathbf{R} = (z\,,\,\mathbf{r})$. The wave functions ϕ_n are determined by the actual shape of the potential well. These functions will be assumed normalized,

$$\int \phi_n^*\,(z)\;\phi_n\,(z)\;\mathrm{d}z \;=\; 1\;. \tag{1.2}$$

The electron energy in the state $\Psi_{n,\,s,\,\mathbf{k}}$ is given by

$$E_{n,\,s,\,\mathbf{k}} = E_n + \frac{\hbar^2\mathbf{k}^2}{2m} \;\equiv\; E_n + E_{\mathbf{k}}\;, \tag{1.3}$$

assuming no magnetic field, so that the spin degeneracy is not lifted.

At a finite temperature T, the electron density per unit volume is given by

$$\begin{aligned}\rho\,(\mathbf{R}) \;&=\; \sum_{n,\,s,\,\mathbf{k}}^{\text{occ. states}} \Psi^*_{n,\,s,\,\mathbf{k}}\,(\mathbf{R})\;\Psi_{n,\,s,\,\mathbf{k}}\,(\mathbf{R}) \\ &=\; \sum_{s\,,\,n}\int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2}\;\Psi^*_{n,\,s,\,\mathbf{k}}\,(\mathbf{R})\;\Psi_{n,\,s,\,\mathbf{k}}\,(\mathbf{R})\;f\,(E_{n,\,s,\,\mathbf{k}} - E_{\mathrm{F}})\;,\end{aligned} \tag{1.4}$$

where E_{F} is the Fermi level, and $f\,(E)$ is the Fermi function,

$$f\,(E) \equiv \frac{1}{1 + \mathrm{e}^{E/kT}} = -\,\frac{\mathrm{d}\Phi}{\mathrm{d}E}\;, \qquad \Phi \equiv kT\;\ln\left[1 + \mathrm{e}^{-\frac{E}{kT}}\right]. \tag{1.5}$$

Performing the angular integration and the summation over s in Eq. (1.4), we have

$$\begin{aligned}\rho\,(\mathbf{R}) \;&=\; \sum_n\;|\,\phi_n\,(z)\,|^2\;\frac{m}{\pi\hbar^2}\int_0^\infty \mathrm{d}E_{\mathbf{k}}\;f\,(E_{\mathbf{k}} + E_n - E_{\mathrm{F}}) \\ &=\; \sum_n\;|\,\phi_n\,(z)\,|^2\;\frac{m}{\pi\hbar^2}\;\Phi\,(E_n - E_{\mathrm{F}})\;.\end{aligned} \tag{1.6}$$

Let us specialize to the case of one subband only. This means that subbands corresponding to $n = 1, 2, \cdots$ are not occupied, $E_n - E_F \gg kT$. The electron density is given by:

$$\rho(\mathbf{R}) = \rho_0(z, \mathbf{r}) = |\phi_0(z)|^2 \frac{m}{\pi\hbar^2} \Phi(E_0 - E_F) . \tag{1.7}$$

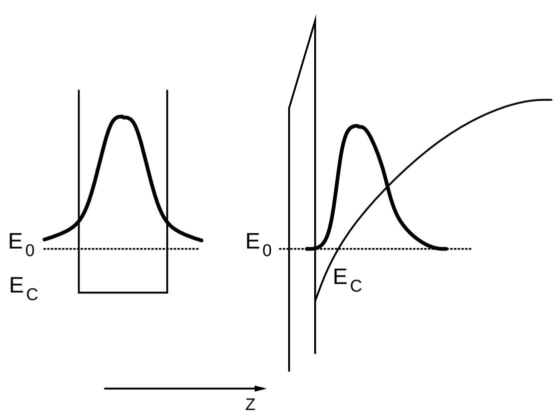
The z dependence in (1.7) is sharp, while the $\mathbf{r}$ dependence is (strictly speaking) absent. However, we can assume that ρ depends "adiabatically" on the position within the plane of 2DEG – through the dependence on $\mathbf{r}$ of the difference $E_F - E_0 \equiv \hbar^2 k_F^2 / 2m$, where k_F is the Fermi wavenumber of the lowest subband. The adiabatic assumption means that the characteristic length $(\nabla \ln k_F)^{-1}$ of in-plane variations in k_F is much larger than the localization length in the z direction.

In a variety of problems we are not really interested in an accurate determination of $\rho(z)$, only in the smooth variation of $\rho(\mathbf{r})$. In this case, we are free to assume a narrow quantum well of thickness d and unspecified shape, and take $|\phi(z)|^2 = 1/d$ inside the well and 0 outside.

The sheet density n (per cm^2) is found from (1.7) and (1.2):

$$n = \int_0^\infty \rho_0(z, \mathbf{r})\, \mathrm{d}z = \frac{m}{\pi\hbar^2} \Phi(E_0 - E_F) , \tag{1.8}$$

valid for an arbitrary position of E_F relative to E_0 **(Part c)**.



Consider the limiting behavior of function $\Phi(E)$:

$$\Phi \equiv kT \ln \left[1 + e^{-\frac{E}{kT}} \right] .$$

where $E \equiv E_0 - E_F$.

$$\Phi = kT\, e^{-\frac{E}{kT}} \qquad \text{for } E \gtrsim kT$$

$$\Phi = -E \qquad \text{for } E \ll -kT$$

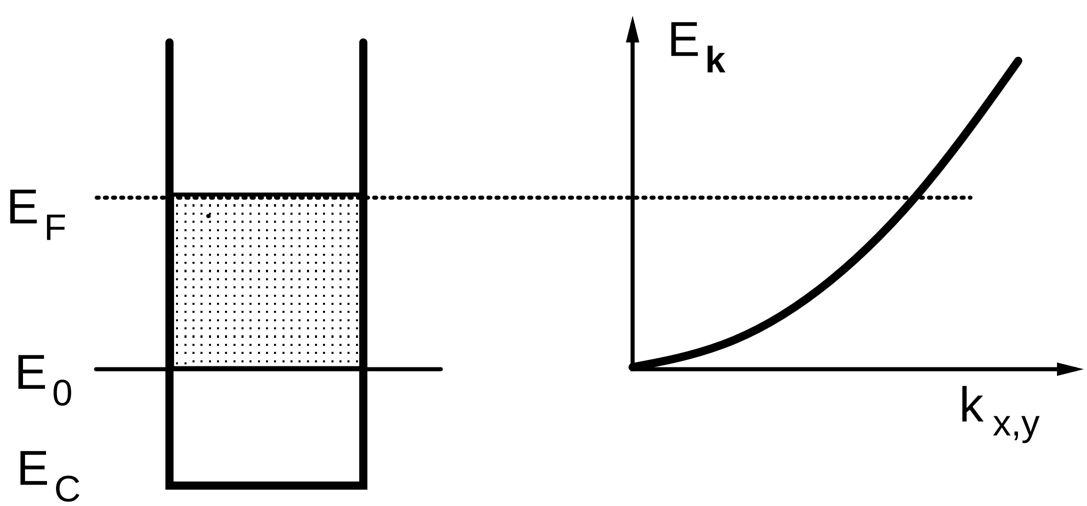
(a): Nondegenerate case, $E_0 - E_F \gtrsim kT$,

$$n = \frac{m\, kT}{\pi \hbar^2}\, e^{-\frac{E}{kT}}$$

The quantity $N_C \equiv \frac{m\, kT}{\pi \hbar^2}$ is the DOS in a 2D subband.

(b): (Strongly) degenerate case, $E_F - E_0 \gg kT$,

$$n = \frac{m\,(E_F - E_0)}{\pi \hbar^2}$$



Problems we intend to discuss today in class are:

- 1-6-4 (Fermi statistics in 2d EG)
- 2-2-1 (pin diode)
- 2-3-4 (pn junction, injection in a narrow layer)
- 2-6-1 (small-signal admittance of a forward pn diode)
- 2-6-2 (CV profiling)
- 2-6-3 (minority carrier storage)
- 2-7-2 (absence of a QM transmission for a heavy object)
- 2-7-3 (electron tunneling estimate)
- 2-10-3 (CV determination of a Schottky barrier height)
- 3-1-4 (floating base npn transistor, IV characteristics)
- 3-1-5 (floating base pnp transistor)
- 3-5-2 (exponential doping in the base; "drift" transistor)

Next week: Mid Term Test: (no class, work at home)

— Problems will be posted on the home page

— 3 hours, time yourself.

— Hand in neat and complete solutions before noon Friday (in a sealed envelope) to Mrs Maria Krause (Chair’s office, ECE Dept)

— indicate how much of each problem you were able to finish within the alotted time

— Mark yourself honestly: your honesty will not be used against you (the 5th Amendment)

— 10 problems, 10 points each: mark yourself

— All problems from: M. S. Shur, *Physics of semiconductor devices*, Prentice Hall, Englewood Cliffs (1990).