Ebers-Moll model

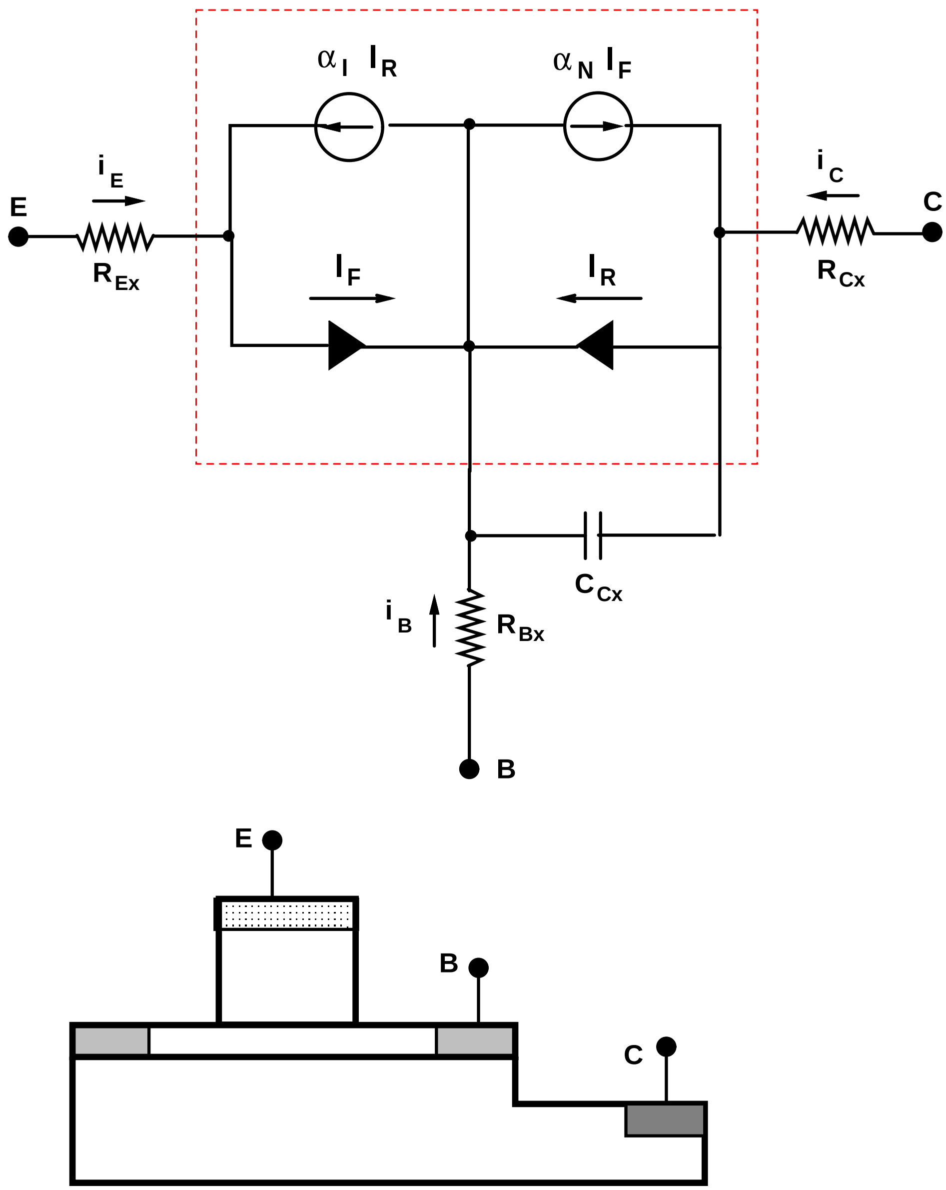

Figure 1: Dashed line indicates the "intrinsic" portion of the device, excluding "parasitic" extrinsic elements.

The model for the diode blocks can be further specified to include the internal junction capacitance.

Small-signal equivalent circuit of an abrupt junction HBT

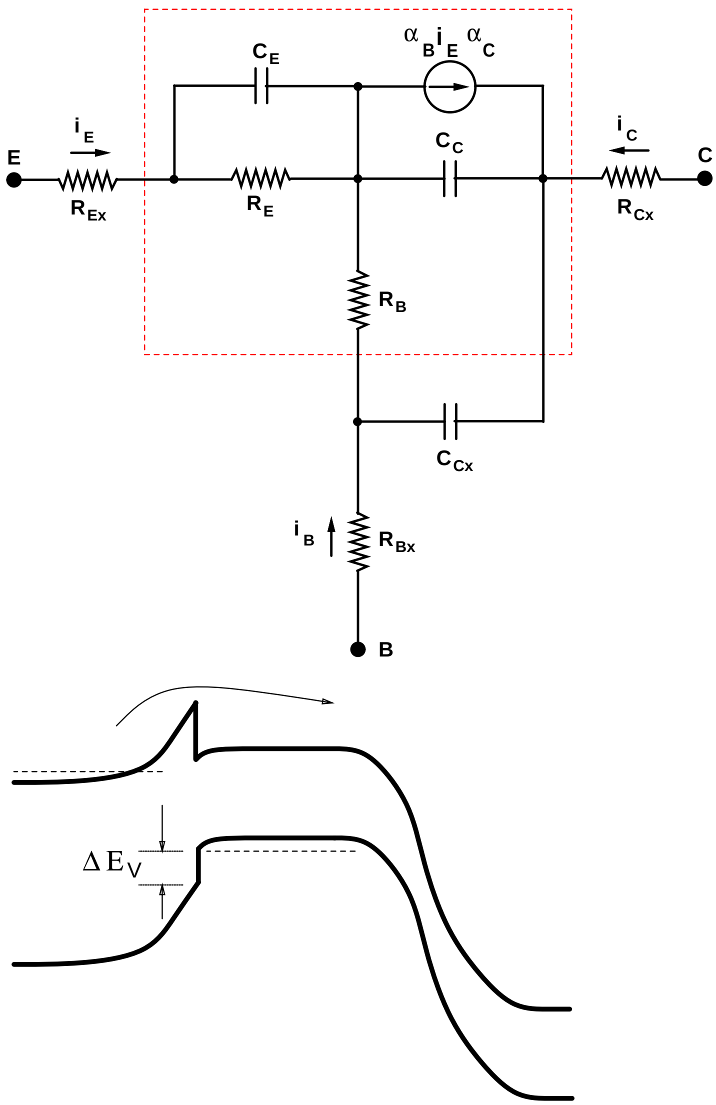

Figure 2: This model is good for "ballistic" propagation of carriers across the base. For diffusive propagation, the intrinsic portion must be adjusted,†

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† A. A. Grinberg and S. Luryi, *IEEE Trans. Electron Devices* **ED-40**, pp. 1512-1522 (1993).

Elements of small-signal analysis

All variables $A(t)$ are considered varying ***harmonically*** in a small range about a dc point:

$$A(t) = A_0 + \delta A \, e^{i\omega t}$$

$$I(t) = I_0 + \delta I \, e^{i\omega t}$$

$$V(t) = V_0 + \delta V \, e^{i\omega t}$$

Many alternative notations, e.g., i and v instead of δI and δV.

The δA 's are ***complex*** quantitites, may be position-dependent fields, $\delta A(\vec{x})$.

The relationship between different δA 's, e.g. between δV and δI (generalized impedances or admittances) depend on the chosen dc point.

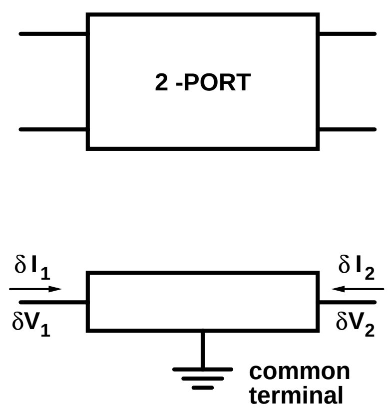

Figure 3: General two-port. Transformers are 2-ports ("passive"). From the small-signal point of view, transistors are two-port amplifiers.

Admittance matrix:

$$\begin{bmatrix} \delta I_1 \\ \delta I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} \delta V_1 \\ \delta V_2 \end{bmatrix}$$

Admittance matrix:

$$\begin{bmatrix} \delta I_1 \\ \delta I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} \delta V_1 \\ \delta V_2 \end{bmatrix}$$

For example:

$$y_{11} \equiv \left[\frac{dI_1}{dV_1} \right]_{V_2} \quad , \text{ input admittance}$$

Impedance matrix:

$$\begin{bmatrix} \delta V_1 \\ \delta V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} \delta I_1 \\ \delta I_2 \end{bmatrix}$$

For example:

$$z_{22} \equiv \left[\frac{dV_2}{dI_2} \right]_{I_1} \quad , \text{ output impedance}$$

Hybrid matrix (h-parameters):

$$\begin{bmatrix} \delta V_1 \\ \delta I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} \delta I_1 \\ \delta V_2 \end{bmatrix}$$

For example:

$$h_{21} \equiv \left[\frac{dI_2}{dI_1} \right]_{V_2} \quad , \text{ forward current gain}$$

Definition of these parameters essentially involves specification of the **boundary condition** at one or another port. Thus

h_{22} is the output admittance for ***open-circuit*** input port.

y_{22} is the output admittance for ***short-circuit*** input port.

h_{21} is the forward current gain for ***short-circuit*** output port, etc.

Each set of parameters (z-parameters, y-parameters, h-parameters) is ***complete*** in the sense that it can be used to derive the other sets unambiguously.

Common terminal configurations

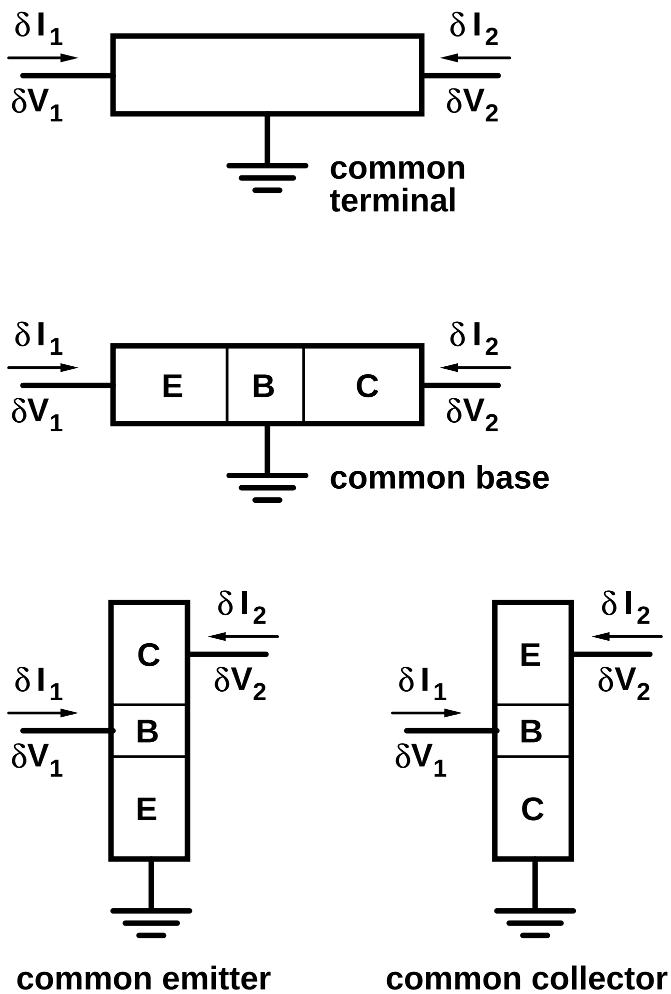

Figure 4: Different common-terminal configurations give rise to very different parameters. Thus, the short-circuit current gains are

$$h_{21}^{e} \equiv \beta \qquad h_{21}^{b} \equiv \alpha$$

and hence

$$h_{21}^{e} = \frac{1 - h_{21}^{b}}{h_{21}^{b}}$$

Indefinite parameters

All of the parameter sets corresponding to different configurations (common-base, common-emitter, common-collector) are derivable from one another.

A convenient trick (works best in y-parameter representation) is to disregard the common reference and treat the third terminal as an additional port:

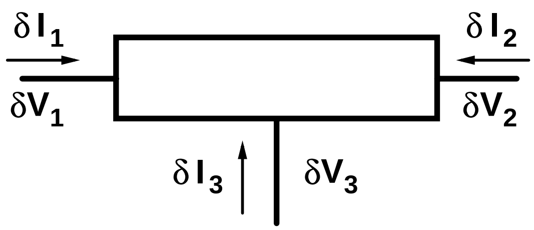

Figure 5: indefinite matrix:

$$\begin{bmatrix} \delta I_1 \\ \delta I_2 \\ \delta I_3 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix} \begin{bmatrix} \delta V_1 \\ \delta V_2 \\ \delta V_3 \end{bmatrix}$$

From Kirhhoff's Law and the fact that the matrix should work for *arbitrary* set of $\{\delta V_i\}$ it follows that the sum of all columns (or rows) in the indefinite matrix is zero.

Thus, if we assume a short circuit at ports 1 and 2, the fact that the sum of all currents must be zero implies that the $\sum$ of y-parameters in the *third* column vanishes, and so on.

To prove that the $\sum$ must vanish in each *row*, we note that if all three $\{\delta V_i\}$ are equal no ac current can flow at any port.

It is exceedingly simple to transform from one common-terminal configuration to another. Thus, if we know the y-matrix in common base configuration, the corresponding common-emitter matrix is:

$$\begin{bmatrix} \mathbf{y}^{b}_{11} & \mathbf{y}^{b}_{12} & y_{13} \\ \mathbf{y}^{b}_{21} & \mathbf{y}^{b}_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix} \rightarrow \begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix} \rightarrow \begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & \mathbf{y}^{e}_{11} & \mathbf{y}^{e}_{12} \\ y_{31} & \mathbf{y}^{e}_{21} & \mathbf{y}^{e}_{22} \end{bmatrix}$$

Power gain definitions:

- ***Power gain*** G is the ratio of power delivered to the load to power input into the network.

  It depends on both the input and the load circuits.

- ***Maximum available gain*** (MAG) is the maximum gain achievable from a particular transistor without external feedback.

  MAG equals the value of forward gain G which results when both the input and the output are simultaneously ***matched*** in an optimum way. For example, realization of MAG requires that the load resistance be matched to the output resistance $Re\,(z_{22})$.

- ***Unilateral gain*** U is the maximum available power gain of a device after it has been made unilateral by adding a lossless reciprocal feedback circuit. This means that the lossless network around the amplifier (inductances and capacitances) is adjusted so as to set the reverse power gain to zero.

Unilateral gain is *independent*† of common-lead configuration !

The unilateral gain U can be calculated from any of the following equivalent expressions:

$$U = \frac{|z_{21} - z_{12}|^2}{4\,[Re\,(z_{11})\,Re\,(z_{22}) - Re\,(z_{12})\,Re\,(z_{21})]}\;;$$

$$= \frac{|y_{21} - y_{12}|^2}{4\,[Re\,(y_{11})\,Re\,(y_{22}) - Re\,(y_{12})\,Re\,(y_{21})]}\;;$$

$$= \frac{|h_{21} + h_{12}|^2}{4\,[Re\,(h_{11})\,Re\,(h_{22}) + Im\,(h_{12})\,Im\,(h_{21})]}\;,$$

where z_{ij}, y_{ij}, and h_{ij} are the impedance, the admittance, and the hybrid parameters of a transistor, respectively, ***for any configuration.***

† This remarkable result (Mason's theorem) is the main reason for the wide-spread use of U. See S. J. Mason, "Power gain in feedback amplifiers", *IRE Trans. Circuit Theory* **CT-1**, pp. 20-25 (1954).

Small-signal model of an abrupt junction HBT

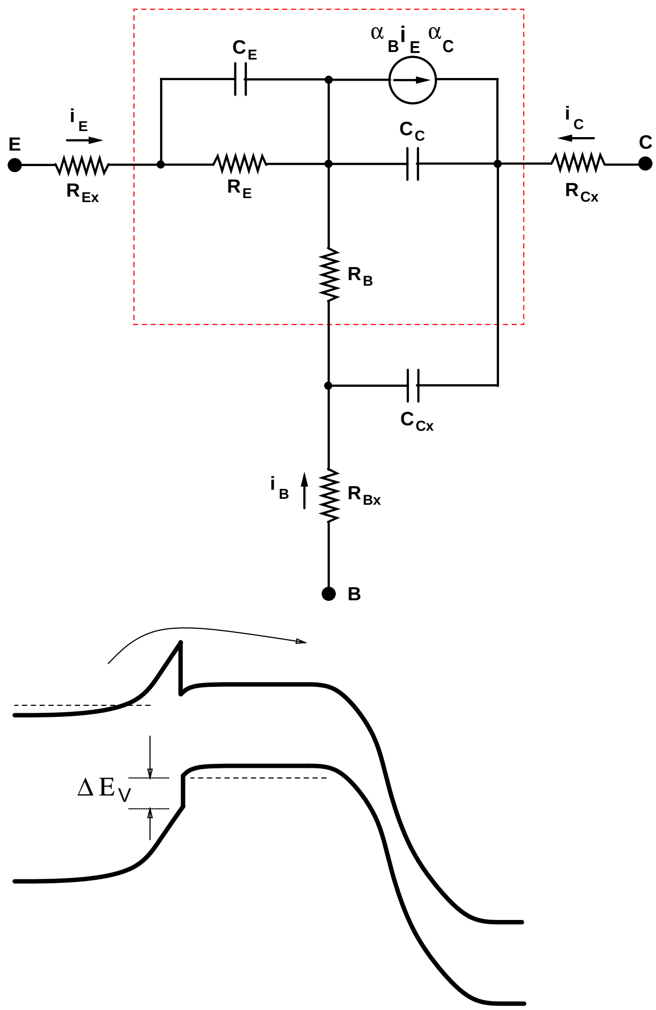

Figure 6: Small-signal analysis of this simple model, including frequency dependence of the power gain in both ballistic and diffusive regimes, has been carried out by Grinberg and Luryi (1993).†

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† A. A. Grinberg and S. Luryi, "Coherent transistor", *IEEE Trans. Electron Devices* **ED-40**, pp. 1512-1522 (1993).

A. A. Grinberg and S. Luryi, "Dynamic Early effect in heterojunction bipolar transistors", *IEEE Electron Device Lett.* **EDL-14**, pp. 292-294 (1993).

Quasi-static (Ebers-Moll-like) model of abrupt-junction HBT can be found in
A. A. Grinberg and S. Luryi, "On the thermionic-diffusion theory of minority transport in heterostructure bipolar transistors", *IEEE Trans. Electron Devices* **ED-40**, pp. 859-866 (1993).

Base transport factor

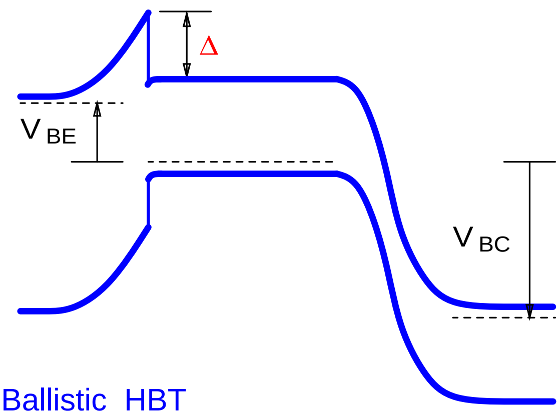
$$\alpha(\omega) = I_c / I_e = |\alpha|\, e^{-i\omega\tau}$$

spirals clockwise with increasing frequency

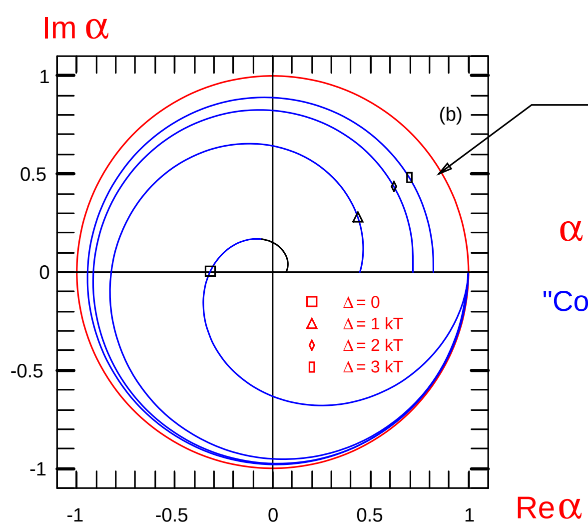
In the usual determination of short circuit current gain f_T degradation of the magnitude $|\alpha|$ plays little (if any) role

For us
it is crucial !

"coherent" base transport when $|\alpha| \gtrsim 0.5$
for $\omega\tau \gtrsim 1$



Ballistic HBT



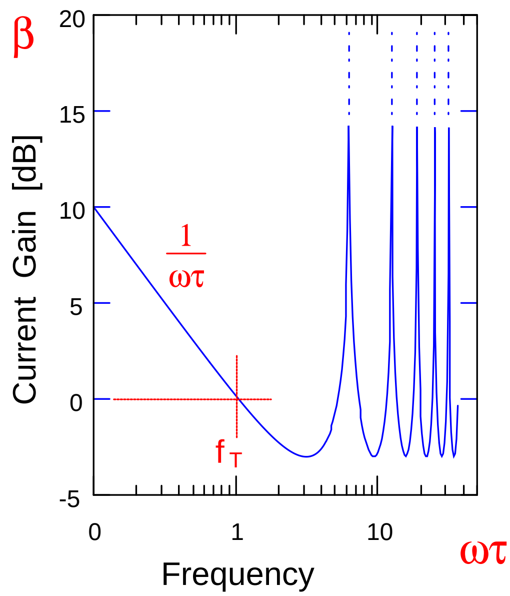
collimated and monoenergetic beam

$$\alpha(\omega) = e^{-i\omega\tau}$$

"Coherent" transport

$\Delta >> kT$

after Grinberg & Luryi, 1993



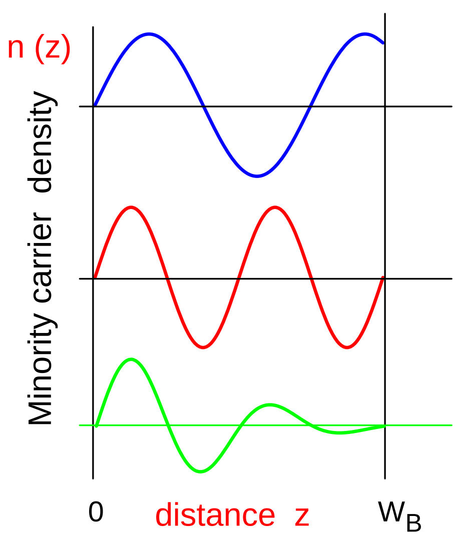
Coherent transistor

$$\alpha(\omega) = e^{-i\omega\tau}$$

$$\beta = \frac{\alpha}{1-\alpha}$$

$$= \frac{1}{2\ \sin \omega\tau/2}$$

$$\approx \frac{1}{\omega\tau} \quad (f < f_T)$$



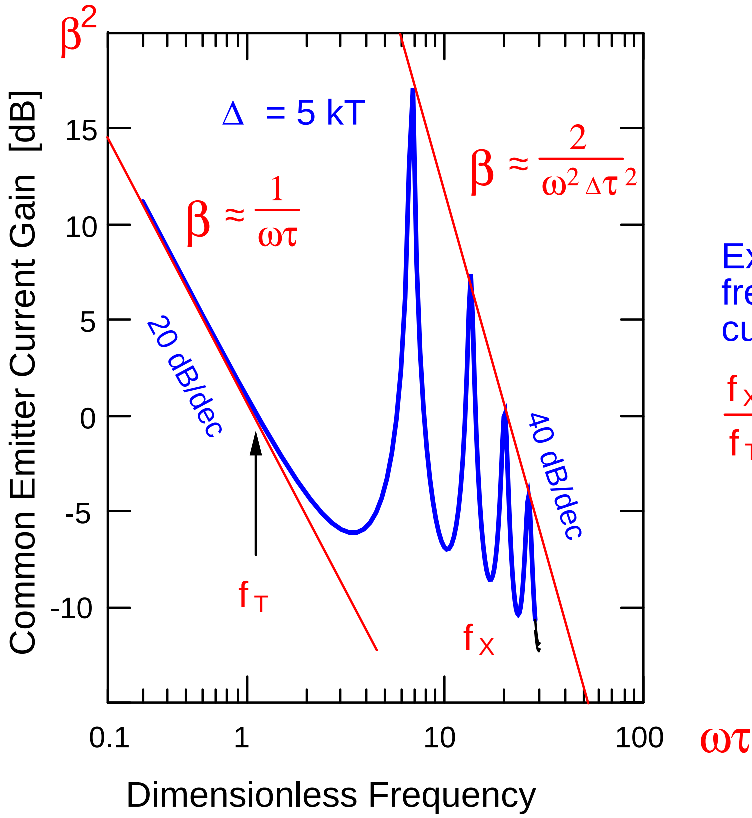
$$n(z) = e^{i\omega(t - z/v)}$$

$$\lambda = 2\pi v / \omega$$

Resonance when
$W_B = m\lambda$
equivalent to
$\omega\tau = 2\pi m$

Landau damped

Example: partial coherence



Extended frequency cutoff

$$\frac{f_X}{f_T} = \frac{\tau}{\Delta\tau}$$

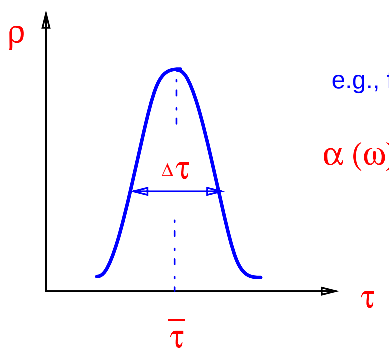
Coherent transistor is limited by the dispersion $\Delta\tau$ rather by τ itself

Partial coherence

base transit time is a random variable with distribution ρ

$$\alpha(\omega) = \int_0^{\infty} \rho(\tau)\, e^{-i\omega\tau}\, d\tau$$

characteristic function of ρ



e.g., for a normal distribution

$$\alpha(\omega) = e^{-\frac{1}{2}\omega^2 \Delta\tau^2}\, e^{-i\omega\bar{\tau}}$$

for thermal distr.

$$\Delta\tau / \bar{\tau} \approx kT / 2\Delta$$

$$\beta = \frac{\alpha}{1-\alpha}$$

max when $\mathrm{Im}(\alpha) = 0$ and $\mathrm{Re}(\alpha) > 0$

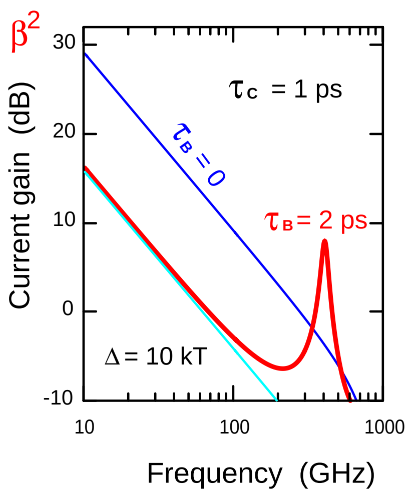
peak frequencies

$$\omega_m \bar{\tau} \approx 2\pi\, m$$

$$\beta_m \approx \frac{2}{\omega_m^2 \Delta\tau^2} \Longrightarrow \omega_m \Delta\tau \approx 1$$

extended frequency cutoff

Inclusion of collector transit



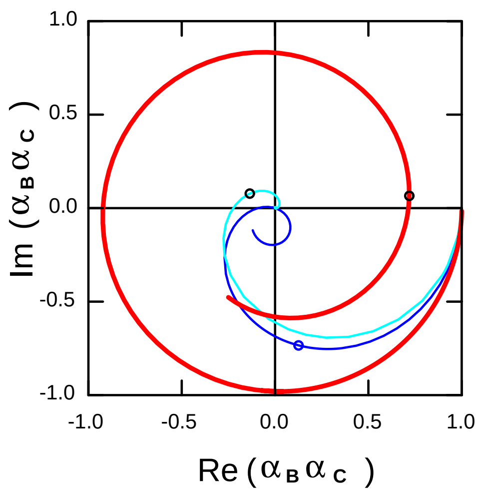
$$\alpha = \alpha_B \alpha_C$$

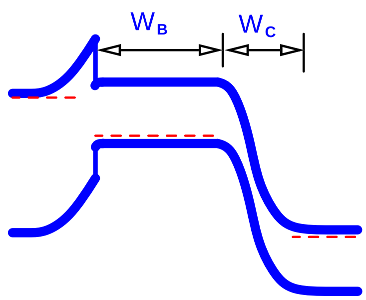
$$\alpha_B = e^{-i\phi}$$

$$\alpha_C = \frac{\sin\theta}{\theta} e^{-i\theta}$$

$$\phi = \omega\tau_B = \omega W_B / V_B$$

$$\theta = \tfrac{1}{2}\omega\tau_C = \tfrac{1}{2}\omega W_C / V_{Sat}$$





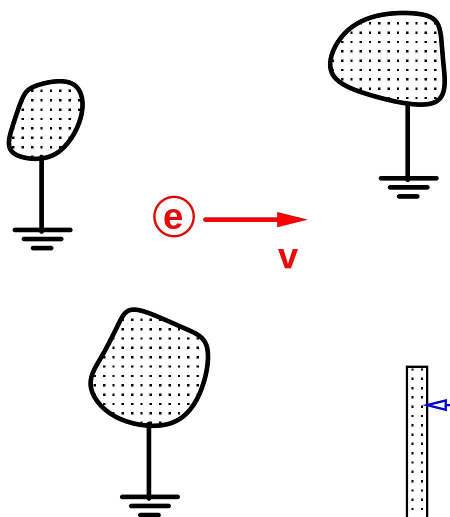
Coherent transistor has current gain at frequencies where the transistor which has no base delay at all is completely damped

What is special about base transit ?

why is the phase gained in constant-velocity collector transit not as good as that gained in base transit ?

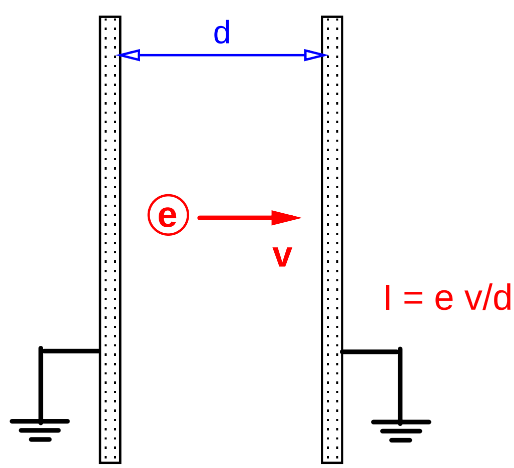
$$\alpha_B = e^{-i\phi}$$

$$\alpha_C = \frac{\sin\theta}{\theta} e^{-i\theta}$$



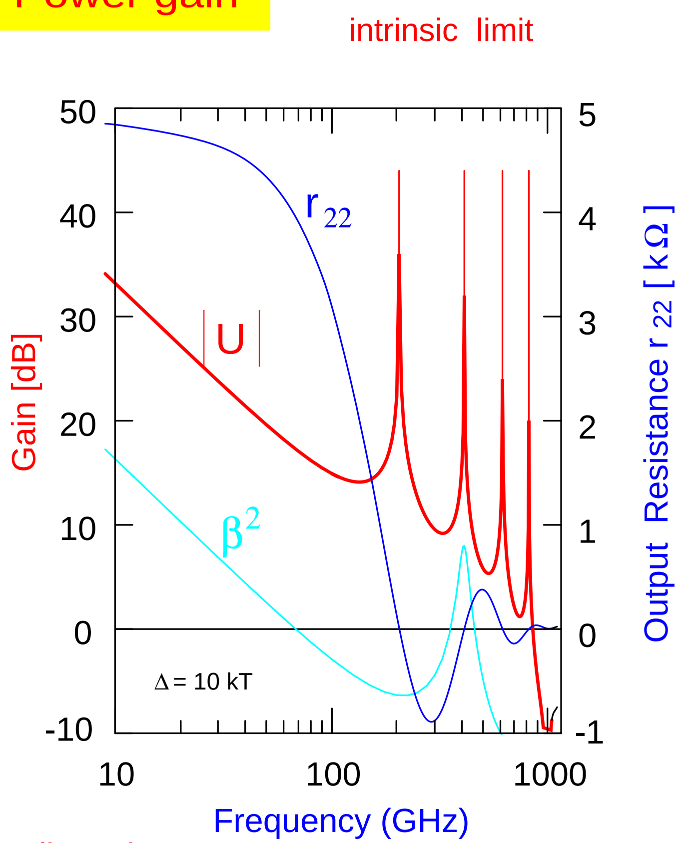
Shockley-Ramo theorem
(Shockley, 1938)

$$I_j = e\ \mathbf{v} \cdot \mathbf{E}_j$$



after Grinberg & Luryi, 1993

Power gain



unilateral gain

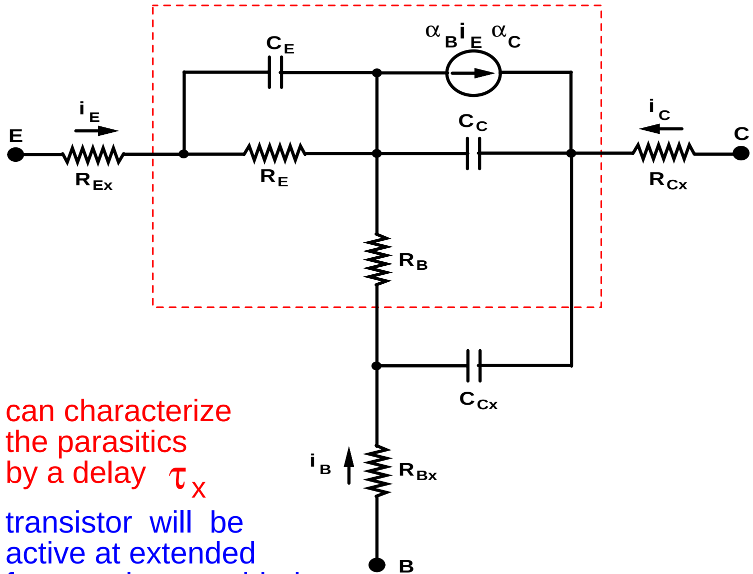
$$U = \frac{|\alpha_B \alpha_C|^2}{4\omega^2 C_C^2 R_B \, r_{22}}$$

$\phi = \omega\tau_B$ $\theta = \omega\tau_C$

$\tau_B = 2$ ps $\tau_C = 1$ ps

$$r_{22} = \frac{\cos(\phi) - \cos(\phi+\theta)}{\omega C_C \, \theta} \, |\alpha_B|$$

Coherent transistor loaded with parasitics



can characterize the parasitics by a delay τ_X

transistor will be active at extended frequencies, provided

$$\left|\alpha_B\right| \sin\left(\phi + \theta/2\right) + \omega\tau_X < 0$$

i.e., provided transistor is not overdamped by the parasitics

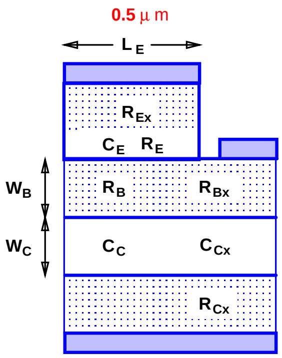
Example: $C_X = 0$

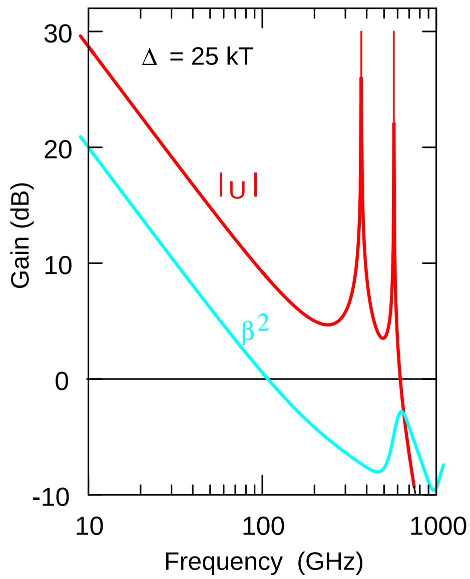
$$U = \frac{\left|\alpha_B \alpha_C\right|^2}{4\,\omega^2 C_C^2 (R_B + R_{Bx})} \; \frac{1}{R_\phi + R_X}$$

$$R_\phi = \frac{\cos(\phi) - \cos(\phi + \theta)}{\omega C_C \;\theta} \; \left|\alpha_B\right|$$

$\tau_X = R_X C_C$ where R_X combination of parasitic resistances

Example: CT loaded with parasitics





R_E = 5 Ω·µm	W_B = 0.1 µm
R_B = 25 Ω·µm	W_C = 0.1 µm
R_{Bx} = 25 Ω·µm	C_C = 0.5 fF/µm
R_{Ex} = 20 Ω·µm	C_E = 10 fF/µm
R_{Cx} = 20 Ω·µm	C_{Cx} = 1 fF/µm
ρ_B = 0.001 Ω·cm	T = 4.2 K

Limitations

Ultra-high frequencies, cryogenic temperatures

$\omega\tau_x < 1$ parasitics

coherence
$kT \ll \Delta < E_{opt}$

impurity scattering

ballistics

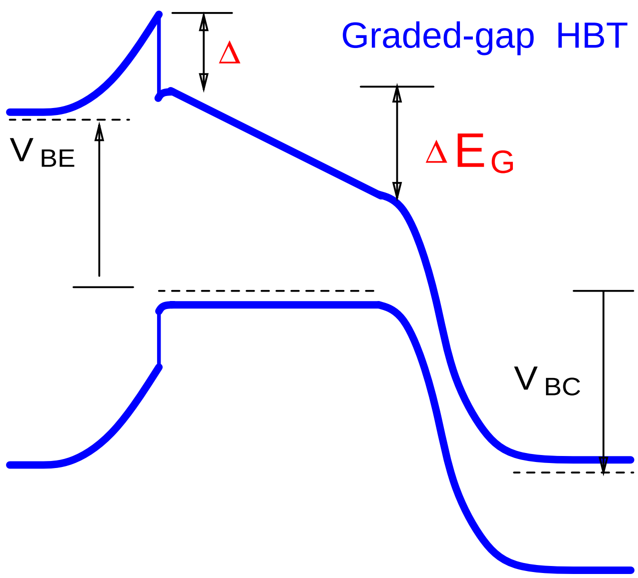
base cannot be too long ($W_B <$ 0.2 µm)

Coherence by other means

Recall: all we need is slow spiralling in of $|\alpha|$

$$\alpha(\omega) = I_c / I_e = |\alpha| \, e^{-i\omega\tau}$$

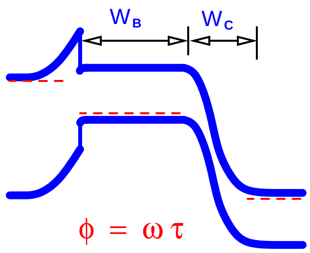
need $|\alpha| \geq 0.5$ for $\phi = \omega\tau \approx \pi$



$$\alpha = e^{-\phi^2/2r} \, e^{-i\phi}$$

where $r = \dfrac{\tau_{diff}}{\tau_{drift}} = \dfrac{\Delta E_G}{2\,kT}$

Coherence by Diffusion

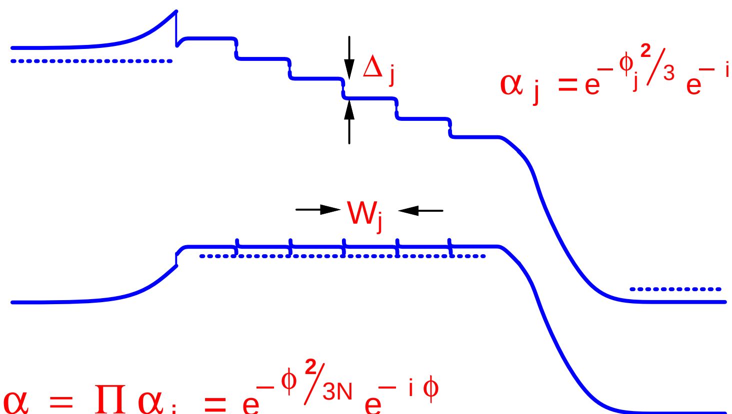


$$\alpha_B = \frac{1}{\cosh\,[2i\,\phi]^{1/2}}$$

$\phi \ll \pi$

$$e^{-\phi^2/3}\; e^{-i\,\phi}$$

to within cubic terms



$$\alpha_j = e^{-\phi_j^2/3}\; e^{-i\,\phi_j}$$

$$\alpha = \Pi\, \alpha_j = e^{-\phi^2/3N}\; e^{-i\,\phi}$$

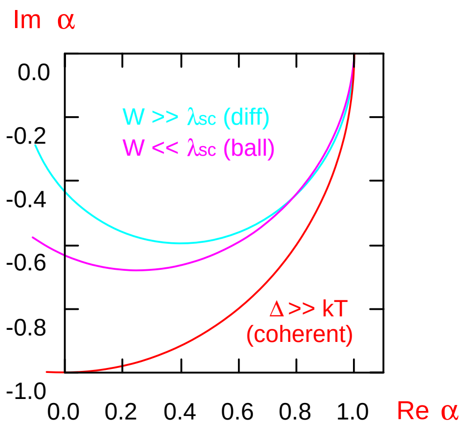
$\phi = \Sigma\, \phi_j = N\, \phi_j$ assuming no return (large enough Δ_j)

For large enough N gain phase ϕ without sacrificing magnitude

$|\alpha| > \pi/2$ at ϕ if $N > \phi^2/3 \ln 2$

For $\phi = \pi$ need $N > 5$

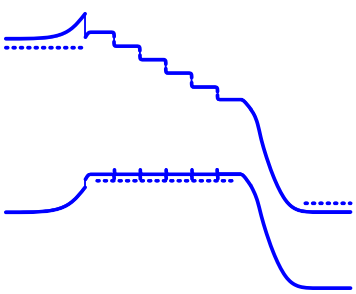
"Stepped up" diffusion

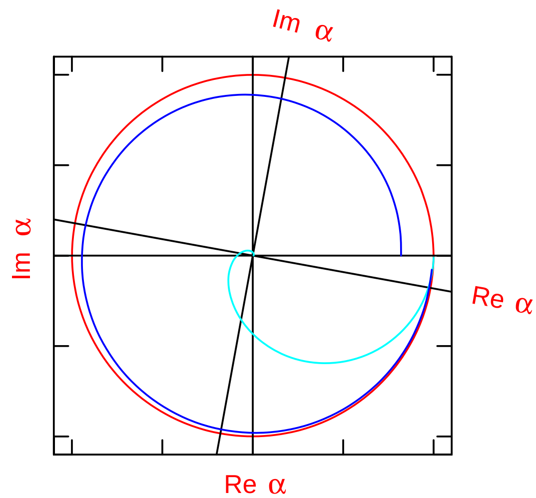


The fact that the magintude of α deviates from unity quadratically in phase φ is true for any transport mechanismm

For diffusion:

$$\alpha_j = e^{-\phi_j^2/3} e^{-i\phi_j}$$



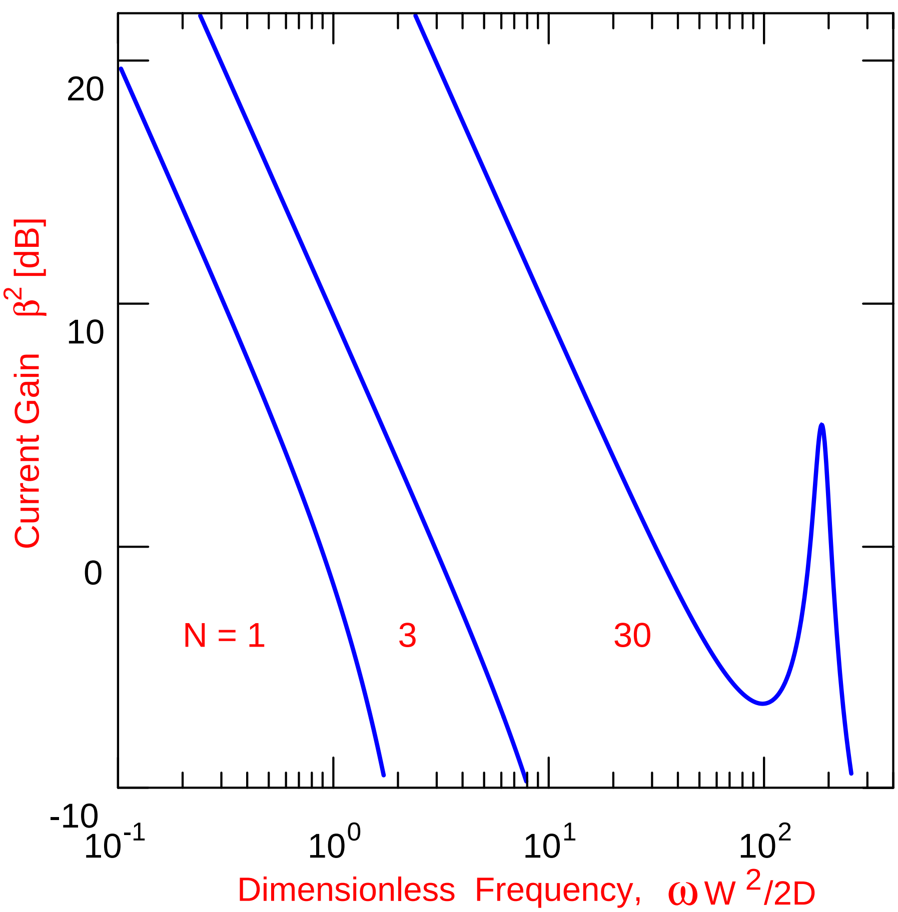


$$\phi = \Sigma \phi_j = N \phi_j$$

$$\alpha = \Pi \alpha_j = e^{-\phi^2/3N} e^{-i\phi}$$

Essential condition: no return at the step

Steps larger than optical phonon energy

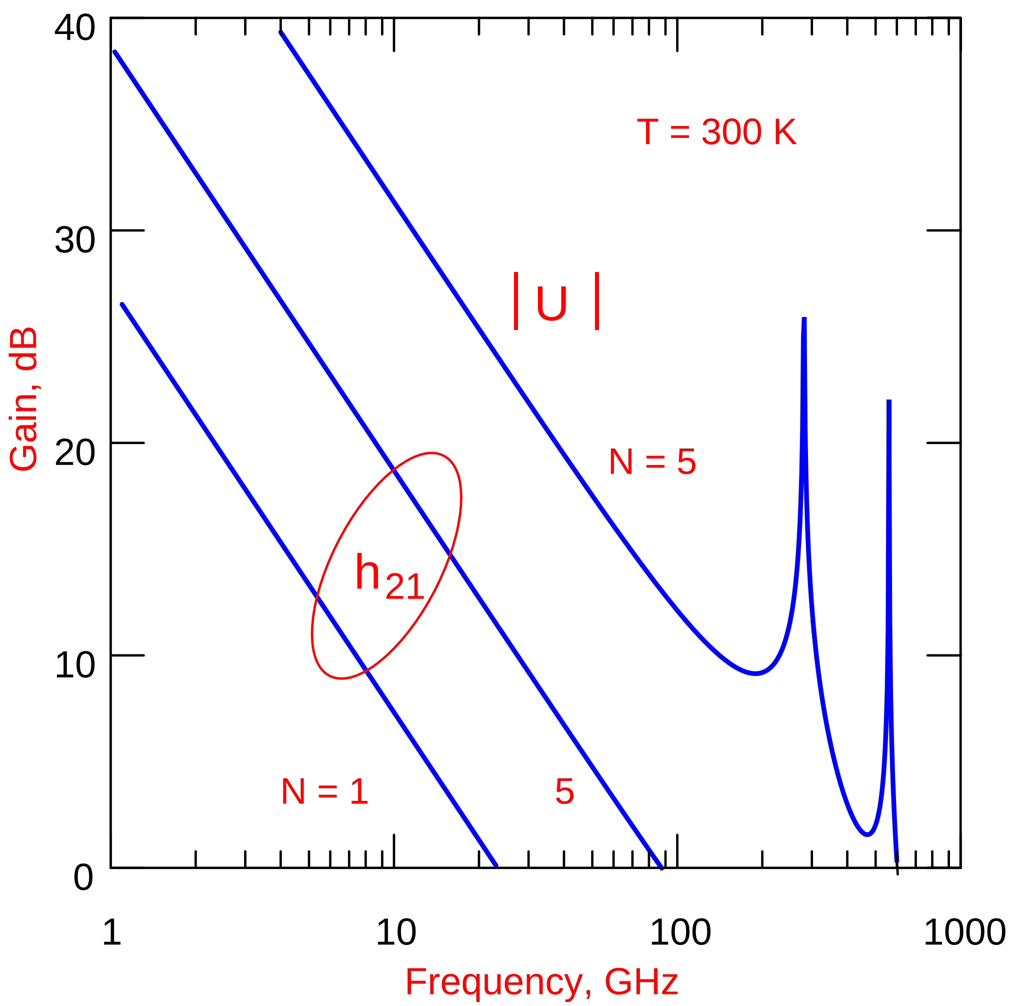


f_T shifts by a factor of N
due to enhancement of effective diffusion velocity

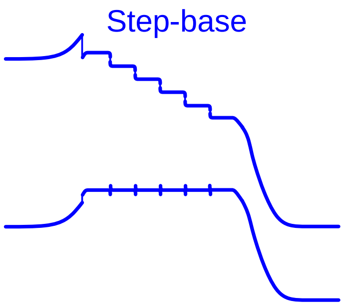
Resonant peak in current gain appears
only for unrealistic values of N > 19

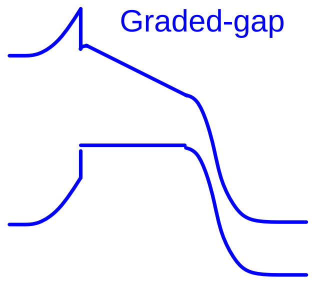
 after Luryi et al., 1993

Exemplary AlGaAs HBT
loaded with parasitics



Comparison





$$\alpha = e^{-\phi^2/3N}\, e^{-i\phi}$$

where $N < \dfrac{\Delta E_G}{E_{opt}}$

For N = 5 in AlGaAs
need $\Delta E_G \gtrsim 180$ meV

$$\alpha = e^{-\phi^2/2r}\, e^{-i\phi}$$

where $r = \dfrac{\tau_{diff}}{\tau_{drift}} = \dfrac{\Delta E_G}{2\,kT}$

Same effect requires r = 7.5
need ΔE_G = 15 kT = 380 meV

Consider W = 2000 A and D = 40 cm^2 / Vs
Peak in U corresponding to the $\phi = \pi$
resonance will occur at the frequency

f = 500 GHz

f = 750 GHz

Summary

Anatoly A. Grinberg & SL
IEEE TED 40, pp. 1512-1522 (August 93)

SL, AAG & Vera B. Gorfinkel
APL 63, pp. 1537-1539 (September 13, 1993)

Coherent ballistic transistor

Nature of gain roll-off with frequency
in collisionless base transport

It is possible to suppress Landau damping
in a cryogenic HBT with abrupt junction

Phaseshift in coherent base propagation
can be used to obtain active transistor
behavior above conventional cutoff frequencies

- Current gain at $f >> f_T$
- Power gain at $f >> f_{max}$
- Role of parasitics

Oscillation frequencies up to 1 THz
are perhaps feasible at low temperatures;
"slow" operation is not possible...
very stringent requirements on the parasitics.

Coherent drift-diffusion transistor

It is possible to slow down and obtain
coherent effects at room temperature
in a graded-gap HBT

Step-base approach appears preferable

Reduction of the parasitic base-collector
capacitance is necessary...
Need: top collector HBT technology