

Shockley diode equation:

$$J = J_{\text{sat}} \left[e^{eV/kT} - 1 \right]$$

$$J_{\text{sat}} = \frac{e D_p p_{n0}}{L_{Dp}} + \frac{e D_n n_{p0}}{L_{Dn}}$$

where $L_{Dp} \equiv \sqrt{D_p \tau_p}$ and $L_{Dn} \equiv \sqrt{D_n \tau_n}$ are diffusion lengths of injected minority carriers (τ_n and τ_p being the minority carrier lifetimes).

Temperature dependence of J_{sat} (in the ideal situation):

$$J_{\text{sat}} \propto n_i^2 \propto e^{-E_g/kT}$$

Ideal Characteristics

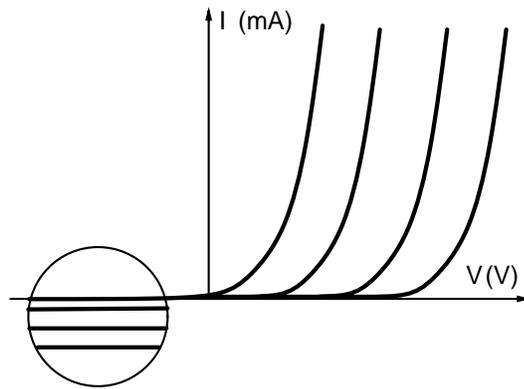


Fig. 2: Current voltage characteristics of a *pn* junction at different temperatures (or different current scales).

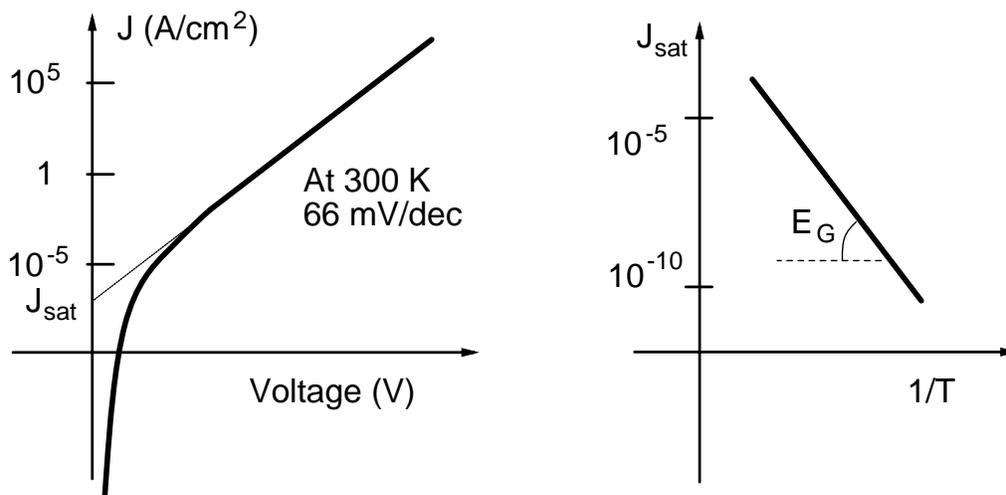


Fig. 3: Current voltage characteristics of a *pn* junction in a semi-log plot; the slope "must" be (but rarely is) equal to e/kT . In reality, often $J \propto e^{eV/(nkT)}$ where $n > 1$ is called the diode "ideality factor".

Arrhenius plot of the saturation current. Activation energy equals E_G .

Non-ideality

- Generation-recombination within depletion region (may dominate at small forward bias). At larger bias, the diffusion current ($n = 1$) will dominate over generation-recombination current ($n \approx 2$).

$$e^{eV/kT} \quad \text{versus} \quad e^{eV/(n kT)} \quad n > 1$$

- Series voltage drop (in the undepleted regions)
- Surface conduction (parallel leakage)
- Reverse breakdown

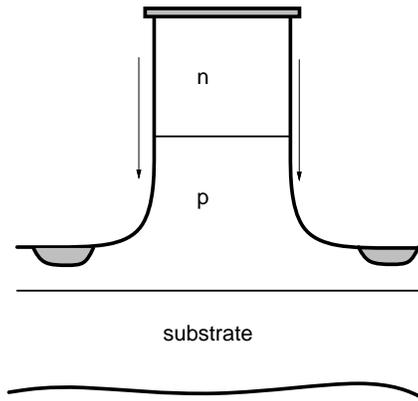


Fig. 4: Surface leakage in an un-passivated mesa diode.

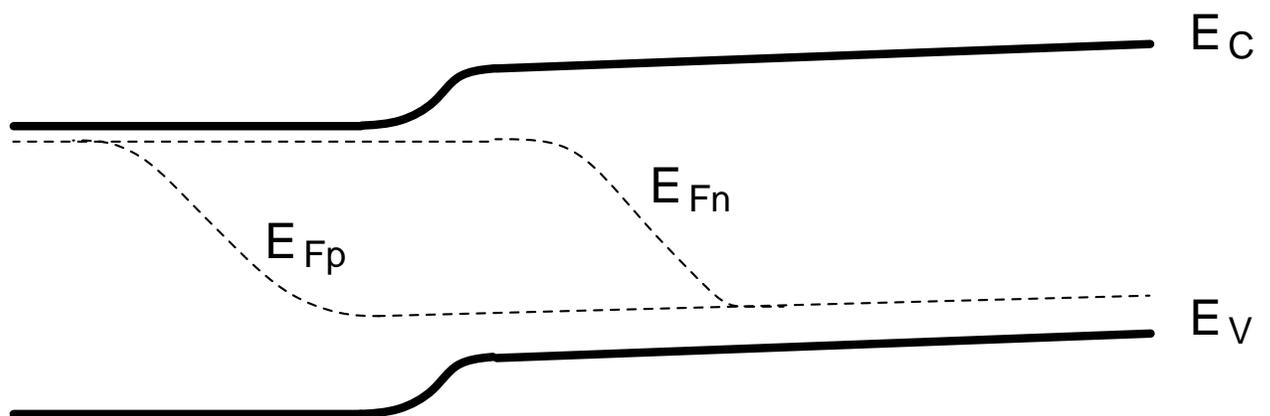


Fig. 5: Series voltage drop at high currents

Digression: Current in conductors

$$\vec{J} = \sigma \vec{E}$$

with a constant conductivity σ . For example, a wire. Where are the charges, responsible for \vec{E} ?

Answer: "outside" the conductor (on its surface).

Consider 2D case.

$$\begin{aligned} \vec{\nabla} \times \vec{E} &= 0 & \leftarrow & \vec{E} = \vec{\nabla} \phi \\ \vec{\nabla} \cdot \vec{J} &= 0 & \leftarrow & \vec{J} = \vec{\nabla} \times \vec{A} \end{aligned}$$

$$E_x = -\frac{\partial \phi}{\partial x} = \frac{\partial A}{\partial y} \quad E_y = \frac{\partial \phi}{\partial y} = -\frac{\partial A}{\partial x}$$

These are the Cauchy-Riemann conditions that $w \equiv \phi - iA$ is **analytic** function of the complex argument $z \equiv x + iy$. This means that $w(z)$ has a definite derivative, independent of the direction in which the derivative is taken. Thus, taking the derivative in the direction x , we find

$$\frac{dw}{dz} = -E_x + iE_y$$

Complex potential w is uniquely determined by the boundary conditions. $w(z)$ is a conformal map of the plane $z = x + iy$ onto the plane $w = \phi - iA$.

The electric field (and current flow) are like in a capacitor.

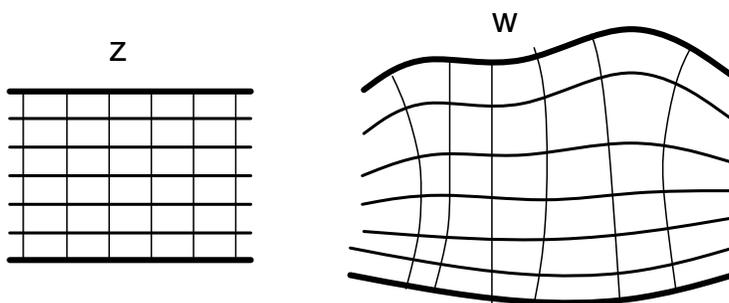


Fig. 6: Equipotential and current (field) lines.

Diffusion capacitance (minority carrier storage)

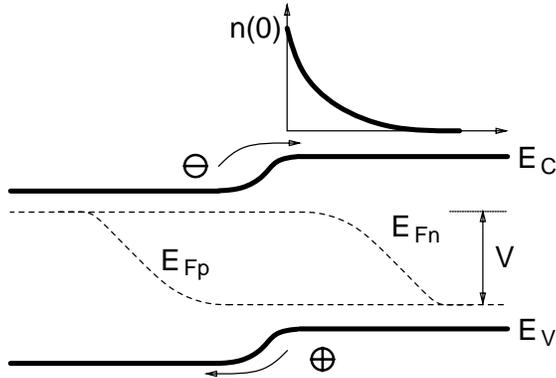


Fig. 7: Injected minority carriers are "stored" for the duration of their lifetime.

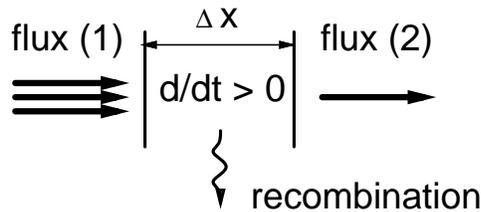


Fig. 8: Continuity equation (time-dependent)

$$\frac{\partial(n \Delta x)}{\partial t} = D \frac{\partial n(1)}{\partial x} - D \frac{\partial n(2)}{\partial x} - \frac{n - n_{p0}}{\tau} \Delta x$$

$$\frac{\partial n}{\partial t} = D \frac{\partial^2 n}{\partial x^2} - \frac{n}{\tau}$$

For harmonic variation, $n(t) = n + \delta n e^{i\omega t}$, where n is the solution of static equation, the variation amplitude δn obeys an equation of the form

$$i \omega \delta n = D \frac{\partial^2 \delta n}{\partial x^2} - \frac{\delta n}{\tau}$$

which is similar to the static equation – with τ replaced by

$$\frac{\tau}{1 + i \omega \tau}$$

The ac behavior of a forward-bias pn junction is similar to a parallel RC circuit, with $RC \approx \tau$.

Recall **depletion capacitance**:

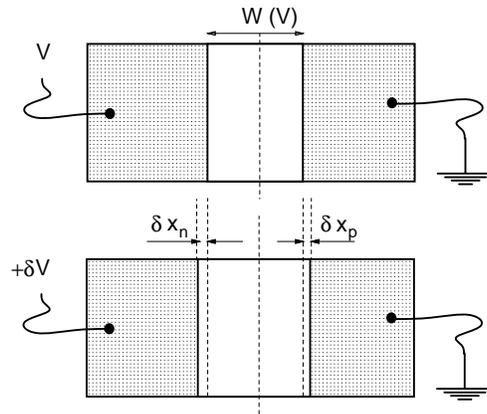
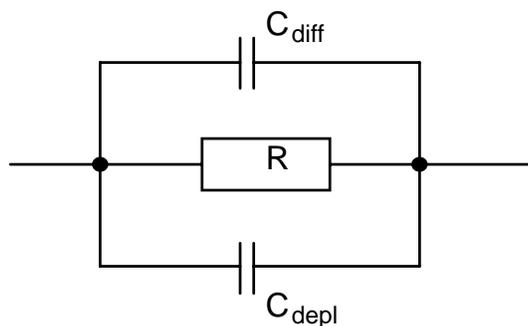


Fig. 9: The depletion capacitance dominates the junction admittance at reverse biases. In forward regime, the depletion and the diffusion capacitances are effectively in parallel to each other and to the junction conductance as well. These parameters are functions of the dc bias:

$$\begin{aligned} C_{\text{depl}} &= C_{\text{depl}}(V) \\ C_{\text{diff}} &= C_{\text{diff}}(V) \\ R &= R(V) \end{aligned}$$

Fig. 10: The simplest equivalent circuit of the *pn* diode is of the form:



Junction Breakdown: Tunneling

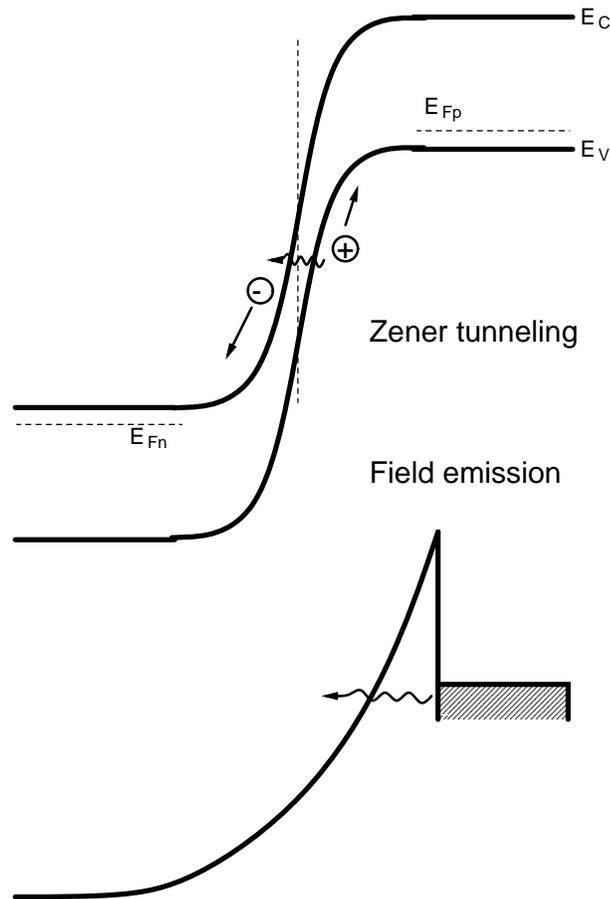


Fig. 11: Tunneling breakdown is not very important in Si *pn* junctions (usually pre-empted by avalanche breakdown); junction can be designed, however, for **early** and rapid onset of breakdown (beating 66 mV per decade in forward turn-on at room temperature). Zener diode is a commercial device, but more often than not it employs avalanche mechanism rather than Zener tunneling. Tunneling breakdown is common for Schottky diodes.

Junction Breakdown: Avalanche

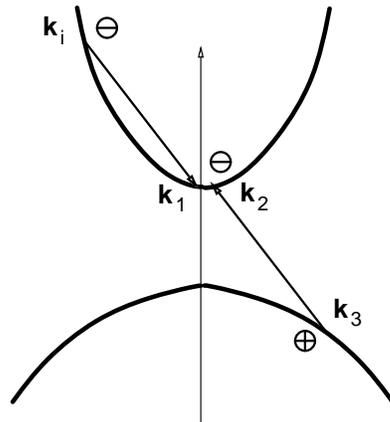


Fig. 12: Impact ionization (inverse process: Auger recombination)

Initial state: one electron at k_i . Final state: two electrons k_1 and k_2 plus one hole at k_3 .

$$k_i - k_1 = k_2 - k_3$$

$$k_i = k_1 + k_2 - k_3$$

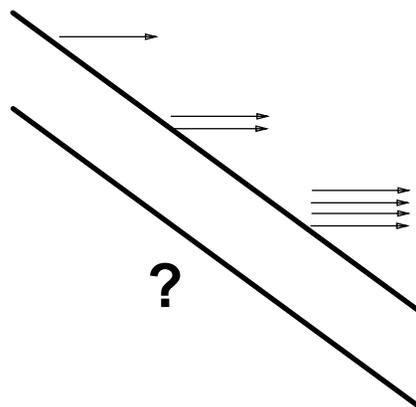


Fig. 13: Avalanche multiplication

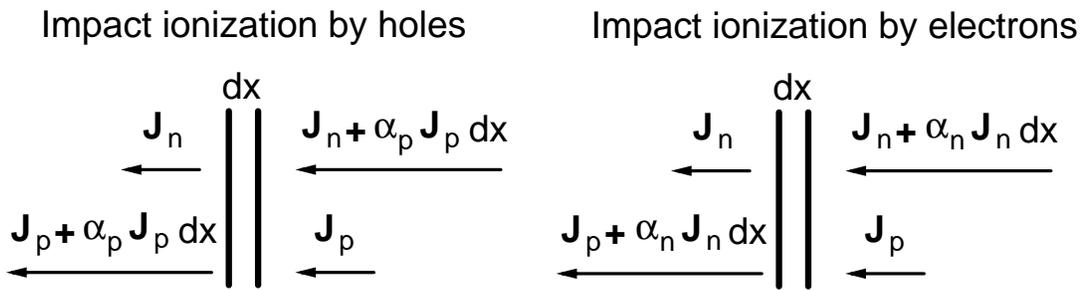


Fig. 14: Avalanche multiplication kinetics

$$\begin{aligned} dJ_n(x) &= \alpha_n J_n(x) dx + \alpha_p J_p(x) dx \\ dJ_p(x) &= -\alpha_n J_n(x) dx - \alpha_p J_p(x) dx \end{aligned}$$

$$\begin{aligned} \frac{dJ_n}{dx} &= \alpha_n J_n(x) + \alpha_p J_p(x) \\ \frac{dJ_p}{dx} &= -\alpha_n J_n(x) - \alpha_p J_p(x) \end{aligned}$$

$$J_n + J_p \equiv J = \text{const}$$

$$\frac{dJ_n}{dx} + (\alpha_p - \alpha_n) J_n(x) = \alpha_p J \quad (*)$$

Solution: Integrate Eq. (*) from 0 to W . In general, the ionization coefficients α 's are functions of position, as they strongly depend on the electric field.

$$J_n(0) \times M_n = J_n(W) \quad (\text{definition of } M_n)$$

$$1 - \frac{1}{M_n} = \int_0^W \alpha_n e^{\int_0^x (\alpha_p - \alpha_n) dx'} dx$$

where α_n and α_p depend on $F(x)$. **Avalanche:** $M_n \rightarrow \infty$

$$1 = \int_0^W \alpha_n e^{\int_0^x (\alpha_p - \alpha_n) dx'} dx$$

Note: same result if take $M_p \rightarrow \infty$, as it should be.

Example: $\alpha_n = \alpha_p \equiv \alpha [E(x)]$

$$M = \frac{1}{1 - \int_0^W \alpha dx}$$

Selected applications of avalanche diodes

- Avalanche photodiodes (APD)
- Transit time oscillators (IMPATT)
- Nonvolatile memory devices (**write**)

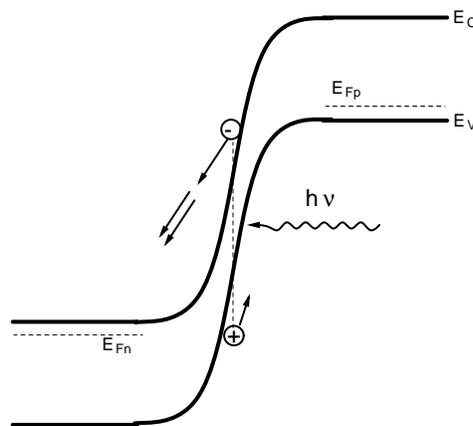


Fig. 15: Avalanche photodiode delivers M electrons to the external circuit per each absorbed photon.

Noise: due to stochastic nature of avalanche multiplication.

mean $\langle M \rangle$ less than dispersion

To minimize noise it is better when $\alpha_n \gg \alpha_p$ (or vice versa) i.e. when avalanche is initiated by one type of carrier.†

† R. J. McIntyre, "Multiplication noise in uniform avalanche photodiodes", *IEEE Transactions on Electron Devices* **ED-13**, 164 (1966);
 R. J. McIntyre, "The distribution of gains in uniformly multiplying avalanche photodiodes", *IEEE Transactions on Electron Devices* **ED-19**, 703 (1972);

APD's and single-photon detection

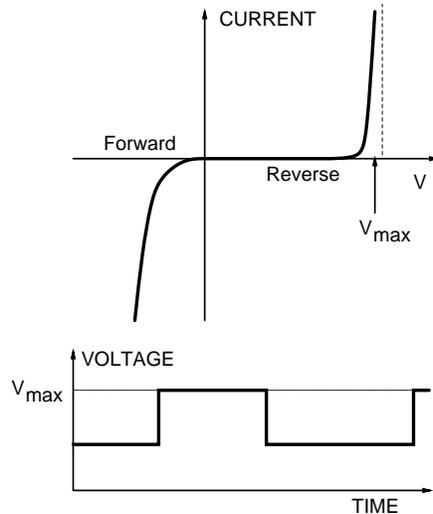


Figure 16: SPAD (single-photon APD) delivers a macroscopic charge (say, 10 mA of current during 1 μ sec) if a photon was absorbed while the diode was biased deep into avalanche. The triggered avalanche is then quenched by a quenching circuit (active or passive).[†]

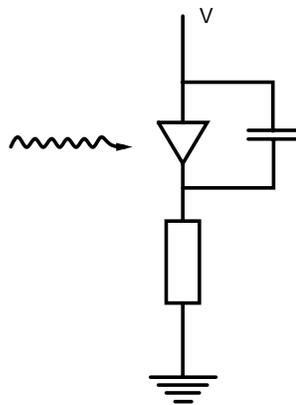


Figure 17: Passive quenching (schematically)

[†] S. Cova et al., "Avalanche photodiodes and quenching circuits for single photon detection", *Applied Optics* 35, pp. 1956-1976 (1996).

Schottky diodes.

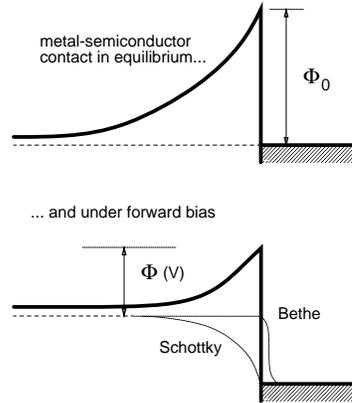


Fig. 18: Band diagram of a Schottky diode.

If the slope is sufficiently steep, the net current is small compared to the oppositely directed and large diffusion and drift fluxes; hence the semiconductor is approximately in equilibrium and Bethe picture applies.

When the slope is gentle, Schottky's diffusion theory applies.†

$$n_{\text{TOP}} = N_C e^{-\beta\Phi(V)} \quad \text{where} \quad \beta \equiv \frac{e}{kT} \quad \text{and} \quad N_C = 2 \left[\frac{m kT}{2\pi \hbar^2} \right]^{3/2}$$

$$v_{\text{AVE}} = \frac{\int_0^{\infty} v_x \rho(v_x) dv_x}{\int_{-\infty}^{\infty} \rho(v_x) dv_x} =$$

$$\Rightarrow J = e n_{\text{TOP}} v_{\text{AVE}} = A T^2 e^{-\beta\Phi(V)},$$

where $A \equiv \frac{e k^2 m}{2 \pi^2 \hbar^3} \approx 120 \times \frac{m}{m_0} \frac{\text{A}}{\text{cm}^2 \text{K}^2}$ is the Richardson constant.

† Transition between these two regimes as the slope decreases with increasing applied voltage is described by a theory of Crowell and Sze, cf. S. M. Sze, *Physics of Semiconductor Devices* (Wiley, 1981) Sect 5.4.3.

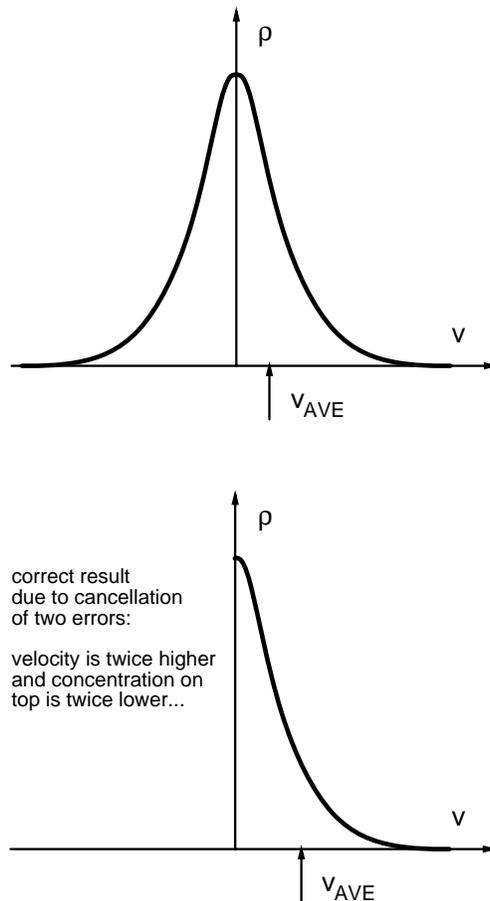


Figure 19: Correct carrier distribution on the top of a Schottky barrier is not an equilibrium Maxwellian distribution but a ***hemi-Maxwellian*** distribution.

The actual carrier concentration is approximately half n_{TOP} given by the equilibrium formula on the previous page, but the correct average velocity is twice v_{AVE} .

This cancellation of two errors means that Richardson's formula is correct.†

† A. A. Grinberg and S. Luryi, "Diffusion in a short base", *Solid St. Electron.* **35**, pp. 1299-1310 (1992).

Schottky barrier lowering.

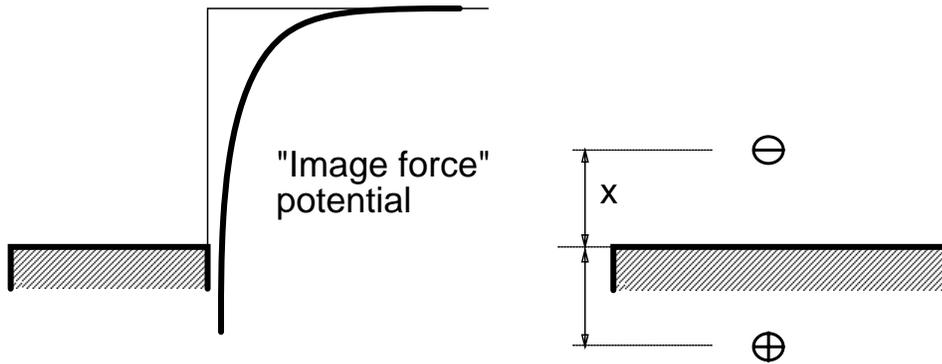


Figure 20: Image force model of the metal work function

$$F(x) = \frac{e}{4\pi\epsilon(2x)^2}$$

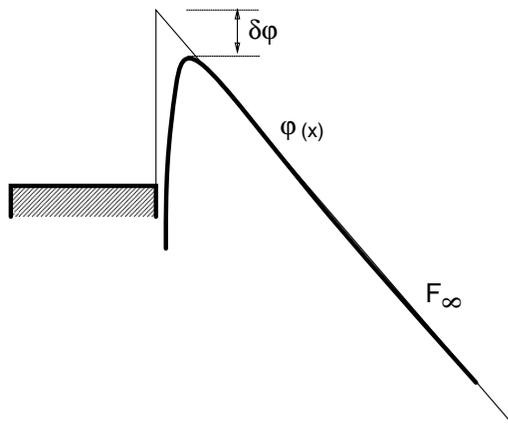


Figure 21: Image force barrier lowering

$$\phi = \frac{e}{16\pi\epsilon x} + F_{\infty} x$$