Polarization Oscillations in Coupled Quantum Wells—A Scheme for the Generation of Submillimeter Electromagnetic Waves

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***Abstract*—In a specially-designed coupled quantum-well (QW) system it is possible to selectively prepare an initial nonstationary electronic state by interband photoexcitation in one of the wells. Excited electrons will oscillate between the two wells giving rise to an oscillating electric dipole moment. In a stacked system of identical QW pairs these dipoles add up producing a tangible alternating voltage which can be used to modulate the incident radiation intensity. With a properly-designed positive feedback loop, connecting the coupled QW system and the light modulator, it is possible, therefore, to generate self-excitation of dipolar oscillations. This would provide a new method of generating submillimeter electromagnetic waves, accompanied by ultrashort optical pulses, periodic at terahertz frequencies.**

I. Excitation of Dipolar Oscillations in a Coupled Quantum-Well System

RECENTLY, an experiment was proposed [1] capable of a direct observation of the time evolution in heterostructure barrier tunneling. The idea of that experiment is to excite a coherent electron oscillation between coupled quantum wells (QW's) and observe the luminescence signal from each well in a distinct frequency range. In the present work this idea is further developed. We shall emphasize the fact that the coherent electronic motion in coupled QW's can manifest itself in ways other than luminescence. In particular, it is accompanied by an oscillating transient polarization, which can be used to generate an oscillating electrical signal.

The idealized structure, illustrated in Fig. 1, contains two quantum wells separated by a heterostructure barrier; the wells have identical ground-state levels ($E_1 = E_2 \equiv E_0$) in the conduction band (labels 1 and 2 correspond, respectively, to the right and the left well). The designed degeneracy of the electronic level is not accompanied by a similar degeneracy in the valence-band quantum wells. Therefore, the interband transition energies $h\nu_1$ and $h\nu_2$ are different in the two wells, which permits us to selectively excite electrons in one or the other well.

In a coupled QW system, electrons will oscillate between the two wells, giving rise to an oscillating luminescence signal with a period directly related to the tunneling time. Indeed, in the presence of tunneling, the levels E_1 and E_2 are not stationary; true eigenstates of an ideal symmetric two-well system are symmetric or antisymmetric with respect to a reflection in the middle plane. To a good approximation, these states can be written

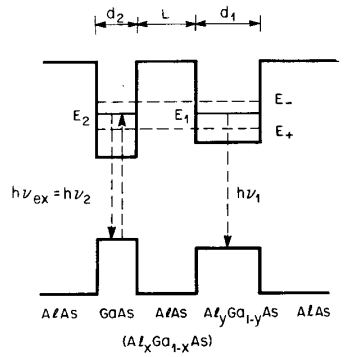

Fig. 1. Schematic band diagram of a proposed structure for direct observation of time evolution in heterostructure barrier tunneling through luminescence oscillation [1]. The idea is illustrated in the instance of a GaAs-AlGaAs heterostructure, although other materials, especially those with a lower electron effective mass, can be used advantageously. The tunnel barrier, separating two quantum wells, can be implemented either as a thin ($L \lesssim 30$ Å) AlAs layer or a slightly thicker $Al_xGa_{1-x}As$ layer with $x \approx 0.4$. One of the wells represents a pure GaAs layer, the other is made of an $Al_yGa_{1-y}As$ alloy with a small fraction of aluminum $y \ll x$. A modified structure, which allows "fine-tuning" of the single-well levels E_1 and E_2 by the electric field of a reverse-biased p–n junction, has been also described [1].

as combinations of the single-well states $|1\rangle$ and $|2\rangle$:

$$|\pm\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle \pm |2\rangle\right)$$

$$\hat{H}_0|\pm\rangle = E_\pm|\pm\rangle, \qquad E_- - E_+ \equiv \hbar\omega > 0. \tag{1}$$

If the excitation energy is tuned to the lower of the two frequencies, $h\nu_{ex} = h\nu_2$, then, immediately upon the excitation, electrons will be "prepared" in state $|2\rangle$, localized in the left well. Subsequent evolution of this state in time is given by

$$|t\rangle = e^{-iE_0t/\hbar}\left[|2\rangle \cos(\omega t/2) - i|1\rangle \sin(\omega t/2)\right] \tag{2}$$

so that the electron densities in wells 1 and 2 oscillate with the frequency ω. In the absence of scattering, the luminescence signals at frequencies ν_1 and ν_2 will oscillate 180° out-of-phase, according to (2). Their intensities will be proportional to $\sin^2(\omega t/2)$ and $\cos^2(\omega t/2)$, respectively. A quasi-classical estimate for the oscillation frequency gives [2]

$$\omega = \frac{\hbar\kappa}{Lm}e^{-\kappa L} \tag{3}$$

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where $\kappa \equiv 2\pi/\lambda$, $\lambda = h/\sqrt{2m(\Phi - E_0)}$ is the de Broglie wavelength of the tunneling electron, and Φ is the heterojunction barrier height.

The oscillatory electronic motion between the wells is analogous to such phenomena as the dipole-moment oscillations of an ammonia molecule used in NH_3 masers [3], energy exchange between coupled optical dielectric waveguides [4], and oscillations of neutral K-mesons between states of different strangeness [5].

Photoexcitation of a nonstationary state is a common phenomenon. It is easy to show that if the excitation duration τ_{ex} is shorter than the period $2\pi/\omega$ of the oscillatory motion, described by (2), then the electron state immediately upon the excitation is a linear combination of the eigenstates of the double-well system, corresponding to a state of the left well. Immediately after the short interaction with an electron, the photon field will not be in a stationary state either, the number of quanta in the interacting mode being no longer sharply determined. Clearly, for our purposes, the excitation should not be too short: if $\hbar/\tau_{ex}$ became larger than $\hbar\nu_2 - \hbar\nu_1$, we would also have an unwelcome excitation of the right-well states. Shortness of the excitation should not be understood literally: in fact, we shall be discussing *continuous* excitation, characterized by a generation rate $G(t)$. In this discussion, it will be left understood that τ_{ex} is the duration of phase coherence in the excitation light signal. This amounts to the assumption that the spectral width of that signal, $\Delta\nu = 1/\tau_{ex}$ satisfies

$$\omega/2\pi \lesssim \Delta\nu \ll \nu_2 - \nu_1. \tag{4}$$

The oscillating polarization of the double-well system is an electrical signal. If a stacked-up set of identical double wells is excited simultaneously, their polarization adds up, producing a potential difference between the top and the bottom layers. In a sense, the sample will behave like a pyroelectric with a variable internal polarization. One purpose of this paper is to draw attention to the possibility of using the electrical signal produced by oscillating electrons to control a valve that would modulate the incident light. That would form a positive feedback loop, quite similar to those used in self-oscillating systems [6].

Possible implementations of the modulator will not be discussed in this work. However, any implementation of the feedback loop requires understanding of the system behavior under various forms of excitation and including relaxation processes. The bulk of this paper is devoted to formulation and solution of an appropriate Bloch-like equation describing the evolution of the system's density matrix and the time-dependent polarization. General solution of the relaxation problem (Section II) is applied first to the case of a finite excitation duration, illustrated in the instance of transient luminescence oscillation (see Section III), and then to the case of a periodically pulsed excitation (Section IV), relevant to the consideration of a self-oscillatory system.

II. General Solution of the Coupled-Well Problem in the Presence of Relaxation and an Arbitrary Time-Dependent Excitation

Dealing with relaxation, it is convenient to describe the system by a density matrix of the form

$$\hat{\rho} = \frac{\rho_0\hat{I} + \boldsymbol{\rho}\cdot\hat{\boldsymbol{\sigma}}}{2} = \frac{1}{2}\begin{pmatrix} \rho_0 + \rho_z & \rho_x - i\rho_y \\ \rho_x + i\rho_y & \rho_0 - \rho_z \end{pmatrix} \tag{5}$$

where $\hat{\sigma}_x$, $\hat{\sigma}_y$, and $\hat{\sigma}_z$ are the Pauli matrices, and $\hat{I}$ is the unit 2×2 matrix. Since the total number of electrons n in the two-well system will be variable in time, we shall normalize the density matrix as follows: $\mathrm{Tr}\,\hat{\rho} = \rho_0 \equiv n(t)$ [7].

States, characterized by density matrices proportional to $(1 \pm \hat{\sigma}_j)/2$, are pure states, corresponding to a definite state of polarization along one of the axes x, y, z. Projection operators onto these states are of the form

$$\hat{P}_j = \frac{1 + \hat{\sigma}_j}{2}, \qquad \hat{M}_j = \frac{1 - \hat{\sigma}_j}{2}, \qquad (j = x, y, z). \tag{6}$$

In particular, the pure states with all electrons localized in the left well are described by the density matrices of the form $n\hat{P}_x$ and those in the right well by $n\hat{M}_x$.

We shall consider the time evolution of the electronic system under the influence of a perturbation which generates electrons in the left well only:

$$\hat{V}(t) = G(t)\hat{P}_x \tag{7}$$

with the generation rate $G(t)$ being an arbitrary function of time. Without a loss of generality we can assume that the perturbation is turned on at $t = 0$ (i.e., $G(t) = 0$ for $t < 0$) and that prior to that time there had been no electrons in the conduction band of the two-well system (i.e., $n(t) = 0$ and therefore $\hat{\rho}(t) = 0$ for $t < 0$).

The relaxation processes will be described by the following four phenomenological terms:

$$\hat{R}_{\mathrm{rec}}^{(L)} = \frac{\hat{P}_x\hat{\rho}\hat{P}_x}{\tau_{\mathrm{rec}}^{(L)}} = \frac{(\rho_0 + \rho_x)\hat{P}_x}{2\tau_{\mathrm{rec}}^{(L)}} \tag{8a}$$

$$\hat{R}_{\mathrm{rec}}^{(R)} = \frac{\hat{M}_x\hat{\rho}\hat{M}_x}{\tau_{\mathrm{rec}}^{(R)}} = \frac{(\rho_0 - \rho_x)\hat{M}_x}{2\tau_{\mathrm{rec}}^{(R)}} \tag{8b}$$

$$\hat{R}_{\mathrm{rel}}^{(\mathrm{long})} = \frac{\hat{P}_z(\hat{\rho} - \hat{\rho}^{\mathrm{eq}})\hat{P}_z + \hat{M}_z(\hat{\rho} - \hat{\rho}^{\mathrm{eq}})\hat{M}_z}{T_1} = \frac{(\rho_z - \rho_z^{\mathrm{eq}})\hat{\sigma}_z}{2T_1} \tag{8c}$$

$$\hat{R}_{\mathrm{rel}}^{(\mathrm{tran})} = \frac{\sum_{j=x,j}[\hat{P}_j\hat{\rho}\hat{P}_j + \hat{M}_j\hat{\rho}\hat{M}_j] - \rho_0\hat{I}}{T_2} = \frac{\rho_x\hat{\sigma}_x + \rho_y\hat{\sigma}_y}{2T_2} \tag{8d}$$

where $\tau_{\mathrm{rec}}^{(L)}$ and $\tau_{\mathrm{rec}}^{(R)}$ describe the recombination rate from the left and the right well, respectively (these rates can be substantially different, depending on the equilibrium hole concentrations in the two wells [8]). The relaxation times T_1 and T_2 have a similar meaning to the corresponding times in nuclear magnetic resonance: relaxation of the diagonal elements of the density matrix is described by the longitudinal time T_1 and that of the off-diagonal elements by the transverse time T_2. Relaxation results from the electron scattering by lattice imperfections, impurities, and phonons, as well as from the electron–electron interaction. An expression of T_2 in terms of the matrix elements of the scattering potential is discussed in Appendix A. It is assumed that both T_1 and T_2 are much shorter than the shortest of the two recombination times:

$$\tau_{\mathrm{rec}} \equiv \frac{\tau_{\mathrm{rec}}^{(L)}\tau_{\mathrm{rec}}^{(R)}}{\tau_{\mathrm{rec}}^{(L)} + \tau_{\mathrm{rec}}^{(R)}} \gg T_1 + T_2. \tag{9}$$

This inequality of the time scales for the relaxation and recombination processes permits us to regard the relaxation as proceeding toward a quasi-equilibrium state, described by a

diagonal density matrix $\hat{\rho}^{eq}[n(t)]$ of the form

$$\hat{\rho}^{eq} = \frac{n}{2\cosh(\hbar\omega/2kT)}\begin{pmatrix} e^{\hbar\omega/2kT} & 0 \\ 0 & e^{-\hbar\omega/2kT}\end{pmatrix}. \quad (10)$$

Since the relaxation of off-diagonal elements does not require inelastic scattering processes, it is expected that $T_2 \ll T_1$ but whether or not the strong inequality actually holds is not essential for our purposes.

Time evolution of the density matrix $\hat{\rho}(t)$ is described by the Bloch-type equation:

$$\frac{d\hat{\rho}}{dt} = \frac{1}{i\hbar}[\hat{H}_0, \delta\hat{\rho}] + \hat{V}(t) - \hat{R}_{rec}^{(L)} - \hat{R}_{rec}^{(R)} - \hat{R}_{rel}^{(long)} - \hat{R}_{rel}^{(tran)}. \quad (11)$$

Solution of this equation is obtained most easily by transforming it to the interaction representation, where the equations for ρ_x and ρ_y decouple. For simplicity, we assume that the recombination time constants are equal in both wells, $\tau_{rec}^{(L)} = \tau_{rec}^{(R)}$, independent of time. This assumption requires that the concentration of holes is controlled by the background doping, rather than by photoexcitation. Leaving details to Appendix B, the result is given by

$$\rho_0(t) \equiv n(t) = \int_0^t G(t-t')e^{-t'/\tau_{rec}}\,dt' \quad (12a)$$

$$\rho_x(t) = \int_0^t G(t-t')\cos(\omega t')\,e^{-t'/T_2}\,dt' \quad (12b)$$

$$\rho_y(t) = \int_0^t G(t-t')\sin(\omega t')\,e^{-t'/T_2}\,dt' \quad (12c)$$

$$\rho_x(t) \approx n(t)\tanh(\hbar\omega/2kT)(1 - e^{-t/T_1}). \quad (12d)$$

Assuming that photogenerated holes are stationary within one well, the polarization operator $\hat{D}$ (dipole moment per unit area) is proportional to $\hat{M}_x$:

$$\hat{D} = ea\hat{M}_x \quad (13)$$

where $a \approx Z_{22} - Z_{11} \approx L + (d_1 + d_2)/2$ and $\hat{Z}$ is the electron position operator (we neglect the Stark contribution to the dipole moment due to polarization of the single-well states in an external electric field). In the state $\hat{\rho}(t)$ the time-dependent polarization $D(t)$ is of the form

$$D(t) = ea\,\mathrm{Tr}(\hat{\rho}\hat{M}_x) = (ea/2)[\rho_0(t) - \rho_x(t)]. \quad (14)$$

Before discussing the $D(t)$ resulting from an infinite periodic train of light pulses (Section IV), it is instructive to consider the situation when $G(t)$ is of finite duration. This type of excitation is more relevant to luminescence–oscillation experiments and accordingly we shall discuss it (Section III) in that context.

III. Examples: Transient Luminescence Oscillations

Luminescence signals from the left well $[L^{(L)}(t)]$ and the right well $[L^{(R)}(t)]$ are proportional to the time-dependent electron concentration in the particular well and the recombination rate in that well. As above, we assume that the recombination time constants are equal in both wells, $\tau_{rec}^{(L)} = \tau_{rec}^{(R)} = 2\tau_{rec}$, independent of time. Thus we can write

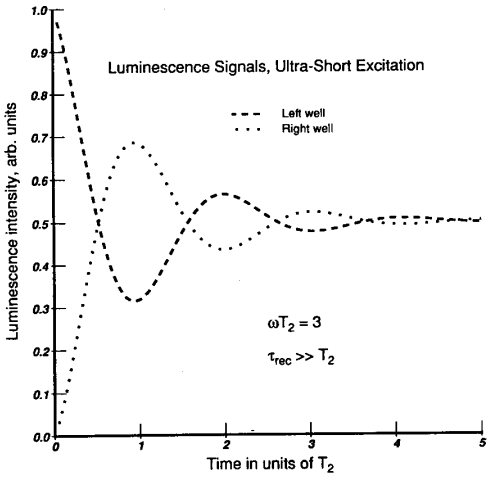

Fig. 2. Decay of the luminescence induced by an ultrashort pulse at $t = 0$. Effects due to recombination are not evident in the figure because of the large assumed τ_{rec}.

$$L^{(L)}(t) = \frac{1}{\tau_{rec}}\mathrm{Tr}(\hat{\rho}\hat{P}_x) = \frac{\rho_0(t) + \rho_x(t)}{2\tau_{rec}} \quad (15a)$$

$$L^{(R)}(t) = \frac{1}{\tau_{rec}}\mathrm{Tr}(\hat{\rho}\hat{M}_x) = \frac{\rho_0(t) - \rho_x(t)}{2\tau_{rec}}. \quad (15b)$$

The right-well signal (15a) is proportional to $D(t)$, cf. (14).

A. Free Precession Decay

Consider the evolution of the system after an infinitesimally short pulse which creates a nonequilibrium polarization by pumping a finite number of electrons in the left well only. Taking in (12),

$$G(t) = n_0\delta(t) \quad (16)$$

we find $\rho_0 = n_0 e^{-t/\tau_{rec}}$, and $\rho_x = n_0 e^{-t/T_2}\cos(\omega t)$, whence

$$L^{(L)}(t) = \frac{n_0}{\tau_{rec}}e^{-t/\tau_{rec}}\left(e^{-t/T_2}\cos^2\frac{\omega t}{2} + \frac{1 - e^{-t/T_2}}{2}\right) \quad (17a)$$

$$L^{(R)}(t) = \frac{n_0}{\tau_{rec}}e^{-t/\tau_{rec}}\left(e^{-t/T_2}\sin^2\frac{\omega t}{2} + \frac{1 - e^{-t/T_2}}{2}\right). \quad (17b)$$

The evolution of polarization in this example is intuitively obvious, and has been discussed previously [1]. In the limit of $\tau_{rec} \to \infty$ it has an exact analogy in NMR: the decay of a free precession of spins about the magnetization axis. Fig. 2 displays the luminescence signals (17) calculated for $\omega T_2 = 3$ and $\omega\tau_{rec} \gg 1$.

B. Excitation Pulse of Finite Duration

Consider now the evolution of a system which is subject to a constant external generation of polarization along the x axis for some period of time t_p, upon which the excitation stops. This corresponds to a $G(t)$ of the form

$$G(t) = \begin{cases} G & 0 < t \le t_p \\ 0 & t > t_p \end{cases} \quad (18)$$

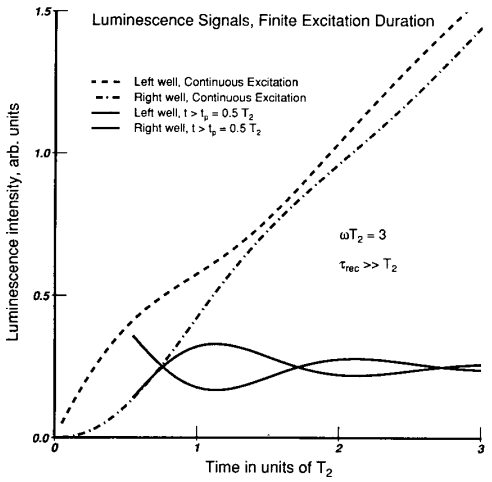

Fig. 3. Luminescence oscillations induced by an extended excitation pulse. Broken lines show the signals during the excitation pulse of indefinite duration. Solid lines show the evolution after the excitation is turned off at $t = 0.5T_2$.

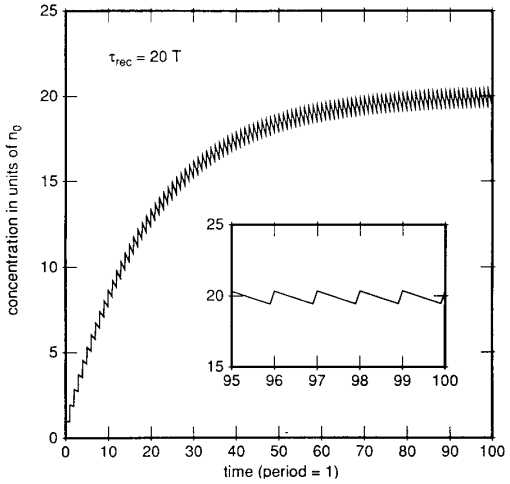

Fig. 4. Evolution of the total electron concentration in a double-well system excited by a train of ultrashort pulses [(22)] of unit period ($T = 1$). Insert shows the established regime ($t \gg \tau_{rec}$) in expanded scale.

which, when substituted in (12), gives

for $t \leq t_p$

$$\rho_0(t) = G\tau_{rec}(1 - e^{-t/\tau_{rec}}) \underset{t_p \ll \tau_{rec}}{\longrightarrow} Gt \tag{19a}$$

$$\rho_x(t) = GT_2 \cos\gamma[\cos\gamma - e^{-t/T_2}\cos(\omega t + \gamma)] \tag{19b}$$

$$\rho_y(t) = GT_2 \cos\gamma[\sin\gamma - e^{-t/T_2}\sin(\omega t + \gamma)] \tag{19c}$$

and for $t \geq t_p$

$$\rho_0(t) = G\tau_{rec}e^{-t/\tau_{rec}}(e^{t_p/\tau_{rec}} - 1) \underset{t_p \ll \tau_{rec}}{\longrightarrow} Gt_p e^{-t/\tau_{rec}} \tag{20a}$$

$$\rho_x(t) = e^{-(t-t_p)/T_2} GT_2 \cos\gamma[\cos(\omega t + \gamma - \omega t_p) - e^{-t_p/T_2}\cos(\omega t + \gamma)]; \tag{20b}$$

$$\rho_y(t) = e^{-(t-t_p)/T_2} GT_2 \cos\gamma[\sin(\omega t + \gamma - \omega t_p) - e^{-t_p/T_2}\sin(\omega t + \gamma)]; \tag{20c}$$

where $\tan\gamma$ is the quality factor of the oscillating system:

$$\tan\gamma \equiv \omega T_2, \qquad (|\gamma| < \pi/2). \tag{21}$$

Expressions (19) and (20) describe the polarization of the system during and after a rectangular excitation pulse of arbitrary duration. Fig. 3 shows the luminescence signals in the limit $\omega\tau_{rec} \to \infty$ calculated for an exemplary relationship between t_p, ω, and T_2. Note that these signals during the excitation do *not* tend to the same value asymptotically, even though the recombination rate is assumed identical in both wells. The fact that we are exciting electrons in the left well only, has the consequence that even as $t_p \to \infty$ the populations in the two wells are different. This implies that one does not require an ultrashort excitation pulse in order to observe the luminescence oscillations—provided the pulse has an abrupt trailing edge. Indeed, in this case the first term in (20b) describes oscillations that decay as $\propto \exp[-(t - t_p)]$, however long has been the pulse duration t_p.

IV. Periodically-Driven Polarization Oscillations

Assume for simplicity a train of δ function excitation pulses, turned on at $t = 0$ and periodic in time with a period T:

$$G(t) = \sum_{\lambda=0}^{\infty} n_0\delta(t - \lambda T). \tag{22}$$

Consider first the evolution of the total electron population $n(t) \equiv \rho_0(t)$, assuming that $n = 0$ initially. Substituting (22) into (12a), we find

$$\rho_0(t) = n_0 \int_0^t \sum_{\lambda=0}^{\infty} \delta(t - t' - \lambda T)\, e^{-t'/\tau_{rec}}\, dt'$$

$$= n_0 e^{-t/\tau_{rec}} \sum_{\lambda=0}^{\infty} \theta(t - \lambda T)\, e^{\lambda T/\tau_{rec}} \tag{23}$$

where $\theta(x)$ is a step function ($\theta = 1$ for $x > 0$ and $\theta = 0$ otherwise). Defining a "residual" (modulo T) time $\tau \equiv t[\text{mod}(T)]$, i.e.,

$$t = \Lambda T + \tau, \qquad \Lambda \text{ integer}, \quad 0 \leq \tau \leq T, \tag{24}$$

we can write

$$\rho_0(t) = n_0 e^{-t/\tau_{rec}} \sum_{\lambda=0}^{\Lambda} e^{\lambda T/\tau_{rec}}$$

$$= \frac{n_0 e^{(T-\tau)/\tau_{rec}}}{e^{T/\tau_{rec}} - 1} - \frac{n_0 e^{-t/\tau_{rec}}}{e^{T/\tau_{rec}} - 1} \underset{t\to\infty}{\longrightarrow} \frac{n_0 e^{(T-\tau)/\tau_{rec}}}{e^{T/\tau_{rec}} - 1}. \tag{25}$$

Evolution of the total electron population $\rho_0(t)$ in a periodically-driven two-well system is illustrated in Fig. 4. As can be expected, it behaves like the charge in a leaky capacitor (with $RC = \tau_{rec}$), periodically boosted by short current pulses. When substituted in (14), the term due to ρ_0 gives rise to a steady-state offset in polarization, owing to slow processes varying on the scale of τ_{rec}. In a circuit, this offset can be shorted out through an inductance—without affecting the faster dynamic processes corresponding to the dynamic component of the den-

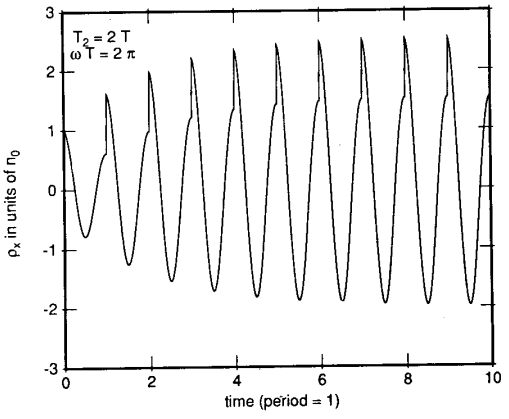

Fig. 5. Evolution of the off-diagonal density-matrix element $\rho_x(t)$ under a train of ultrashort excitation pulses [(22)] whose period is resonant with the characteristic internal period of the system $\omega T = 2\pi$.

sity matrix $\rho_x(t)$ which describes electron oscillation between the two wells with a characteristic internal frequency ω.

Let us now evaluate ρ_x. Substituting (22) into (12b), we find

$$\rho_x(t) = n_0 \int_0^t \sum_{\lambda=0}^{\infty} \delta(t - t' - \lambda T) e^{-t'/T_2} \cos(\omega t')\, dt'$$

$$= n_0 \sum_{\lambda=0}^{\infty} \theta(t - \lambda T)\, e^{(\lambda T - t)/T_2} \cos\left[\omega(\lambda T - t)\right]. \tag{26}$$

Consider first the case of a resonant excitation $\omega T = 2\pi$. In this case, evaluation of the sum (26) is mathematically identical to that in (23) and the result is given by

$$\rho_x(t) = n_0 \cos(\omega t) \frac{e^{(T-\tau)/T_2} - e^{-t/T_2}}{e^{T/T_2} - 1}$$

$$\tau \equiv t \,[\mathrm{mod}\,(T)]. \tag{27}$$

Fig. 5 shows the time evolution of ρ_x, calculated from (27). The steady state, established upon $t \gg T_2$, corresponds to a regime of forced periodic oscillations:

$$\rho_x(t) = n_0 \cos(\omega t)\, F(t), \qquad t \gg T_2 \tag{28}$$

where

$$F(t) = \frac{e^{(T-\tau)/T_2}}{e^{T/T_2} - 1}, \qquad \tau \equiv t \,[\mathrm{mod}\,(T)] \tag{29}$$

is a periodic function of time with an angular frequency ω, an amplitude 1, and a mean value of $(1/2)\coth(T/2T_2)$. The anharmonic distortion brought about by F becomes unimportant in the limit $\omega T_2 \gg 1$, when $F \to T_2/T \gg 1$. In this limit, the oscillatory part δD of the total dipolar polarization reduces to

$$\delta D(t) = -\frac{ean_0}{2} \frac{T_2}{T} \cos(\omega t). \tag{30}$$

Equations (27)–(30) retain their form for a driving force whose period T equals an integral number N of the proper periods of the oscillatory electronic motion $\omega T = 2\pi N$.

We can evaluate $\rho_x(t)$ away from the resonance as well. For an arbitrary ωT, we rewrite (26) in the form

$$\rho_x(t) = n_0 e^{-\tau/T_2} \left[\cos(\omega t) \sum_{\lambda=0}^{\Lambda} e^{-\lambda T/T_2} \cos(\lambda \omega T) - \sin(\omega t) \sum_{\lambda=0}^{\Lambda} e^{-\lambda T/T_2} \sin(\lambda \omega T) \right], \tag{31}$$

where, as before, $\tau \equiv t\,[\mathrm{mod}\,(T)]$. In the established steady-state regime, $t \gg T_2$, we can extend the summation to infinity $\Lambda \to \infty$ and obtain the result in a closed form:

$$\rho_x(t) = \frac{n_0 e^{-\tau/T_2} \left[e^{T/T_2} \cos(\omega t) - \cos(\omega T - \omega t) \right]}{2 \left[\cosh(T/T_2) - \cos(\omega T) \right]}$$

$$\tau \equiv t \,[\mathrm{mod}\,(T)].$$

For $\omega T = 2\pi N$, this expression reduces to (28).

V. Feedback Loop: Requirements for Self-Sustaining Oscillations

Examples described in the preceding sections show that the dynamical response of the double-well system corresponds to a damped harmonic oscillator of characteristic frequency ω and logarithmic decrement $2\pi/\omega T_2$. It is well known [6] that with a positive feedback such systems are capable of self-resonance, i.e., resonance under the action of a periodic force generated by the motion of the system itself.

It is clear that the oscillating polarization of the double-well system is an electrical signal. Consider, for example, a set of stacked double-well structures of identical composition, clad on both sides by conducting layers, as illustrated in Fig. 6(a). Suppose these conducting layers are shorted by a wire. An oscillating polarization, excited by the simultaneous excitation of n electrons, will give rise to an ac current with an amplitude of order $\sim en\omega$. This signal can be used to control a modulator of light incident on the multiple double-well structure.

The feedback loop is schematically illustrated in Fig. 6(b). The optical length between the modulator and the oscillator must be adjusted to time the arrival of the transmitted signal. It is premature to discuss at this point any concrete implementation of the modulator. However, the requirements it must satisfy, are clear. First, it has to operate at frequencies of interest. Therefore, the modulator response time τ_{mod} must satisfy $\omega\tau_{\mathrm{mod}} < 1$. Second, the modulator's transparency must be a sufficiently steep function of the electrical input signal. If that requirement is satisfied, the modulator becomes analogous to a valve with step-like characteristics and the feedback scheme becomes quite similar to those described in [6].

VI. Discussion and Conclusion

We have described a rather exotic scheme for generating electromagnetic oscillations. The generator is powered by a constant optical source and produces submillimeter electromagnetic waves, accompanied by ultrashort optical pulses at terahertz frequencies. The key advantage of the proposed scheme lies in the frequency range of the generation which is hardly accessible to transistor electronics.

It may appear that the frequency limits of the present generation scheme are similar to those of double-barrier resonant-tunneling diodes. Such diodes have been reported recently [9] to produce oscillations at frequencies above 400 GHz. It should

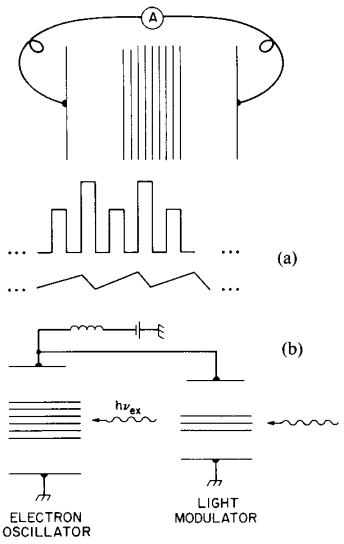

Fig. 6. (a) Multiple stack of identical double-well structures clad by two conducting layers shorted to each other. The internal oscillating polarization is screened by the current flowing in the external circuit. The sawtooth illustrates an electrostatic potential distribution inside the structure at some fixed time. (b) Illustration of a feedback loop. Electrical signal from a multiple stack of identical double-well structures is used to control the transparency of a modulator of incident light. It is assumed that the modulator has a capacitive input. Fine-tuning of the double-well resonators can then be done by applying a constant field through an inductance, as shown. This inductance may be needed also to complete the circuit for slowly-varying (with a characteristic time of τ_{rec}) signals.

be pointed out, however, that the limit frequency f_{max} of double-barrier oscillators scales differently with the tunneling-barrier thickness. In those oscillators, the f_{max} is limited by the inverse lifetime of the resonant state in the quantum well [10] and therefore scales as $\exp(-2\kappa L)$. In the present case, the oscillation frequency (3) scales as $\nu = 2\pi\omega \propto \exp(-\kappa L)$ and, therefore, the same frequency is achievable with barriers which do not have to be as narrow [11].

Appendix A
Physical Meaning of the Relaxation Time T_2

The relaxation time T_2 governs relaxation of the phase difference between the states in the left and the right well and, therefore, it can be referred to as the phase relaxation time. Any process, that randomizes the phase of electronic oscillations between the two wells, shortens T_2. The most important such processes in our problem are due to the elastic scattering of electrons within the QW plane. However, it should be noted that in order to produce the phase relaxation, a scattering potential must affect the transverse motion of states in each well differently. Indeed, a potential, that is a function of the in-plane coordinates x, y only, can be factored out of the solution of the Schrödinger equation.

Evaluation of T_2 for a general scattering mechanism is a difficult problem. This problem simplifies considerably in the first Born approximation, where we can adapt the treatment of Kazarinov and Suris [12], who considered the problem of electron hopping between sites of a superlattice in a strong electric field. Their approach parallels the classical derivation by Kohn and Luttinger [13] of the kinetic equation from a quantum-mechanical model. The transverse relaxation time is obtained in a way similar to the momentum relaxation time that enters the in-plane electrical conductivity. This approach can be adapted virtually without change to the present problem; the result is expressed in terms of the matrix elements $V_{pp'}^{(L,R)}$ of the scattering potential between plane-wave states p and p' of the electronic motion in a given well:

$$\frac{1}{T_2} = \frac{\pi}{\hbar}\sum_{p'} |V_{pp'}^{(L)} - V_{pp'}^{(R)}|^2 \delta(E_p - E_{p'}), \tag{A1}$$

where $E_p \equiv p^2/2m$ is the kinetic energy of the in-plane motion.

Equation (A1) shows that T_2 may be considerably longer not only than the lifetime of a state with a well-defined in-plane momentum but also longer than the transport time of quantum-well electrons. As discussed by Kazarinov and Suris [12], the relaxation is related to the interference of two second-order processes: a virtual transition $Lp \to Lp'$ due to scattering, followed by tunneling $Lp' \to Rp'$ and a virtual tunneling $Lp \to Rp$ followed by scattering $Rp \to Rp'$. If $V^{(L)} = V^{(R)}$, as would be the case for the scattering by distant impurities, then these two amplitudes cancel and there would be no contribution to the phase relaxation from such a potential.

On the other hand, it should be noted that the electron–electron scattering, which gives no contribution to the mobility, does contribute to the phase relaxation like any other short-range scattering.

Appendix B
Solution of the Quantum Relaxation Equation

Written in terms of the components of the density matrix (5), the quantum relaxation equation (11) assumes the usual (slightly modified) form of the Bloch equation of nuclear magnetism:

$$\frac{d\boldsymbol{\rho}}{dt} = \boldsymbol{\rho} \times \boldsymbol{\omega} - \frac{\overset{o}{\boldsymbol{x}}\rho_x + \overset{o}{\boldsymbol{y}}\rho_y}{T_2} - \frac{\overset{o}{\boldsymbol{z}}(\rho_z - \rho_z^{eq})}{T_1} + G(t)\overset{o}{\boldsymbol{x}} - \frac{\overset{o}{\boldsymbol{x}}\rho_x}{\tau_{rec}} - \frac{\overset{o}{\boldsymbol{x}}\rho_0[\tau_{rec}^{(R)} - \tau_{rec}^{(L)}]}{\tau_{rec}^{(R)}\tau_{rec}^{(L)}}; \tag{B1}$$

$$\frac{d\rho_0}{dt} = G(t) - \frac{\rho_0}{\tau_{rec}} - \frac{\rho_x[\tau_{rec}^{(R)} - \tau_{rec}^{(L)}]}{\tau_{rec}^{(R)}\tau_{rec}^{(L)}}, \tag{B2}$$

where $\overset{o}{\boldsymbol{x}}$, $\overset{o}{\boldsymbol{y}}$, and $\overset{o}{\boldsymbol{z}}$ are unit vectors and $\boldsymbol{\omega} \equiv \omega\overset{o}{\boldsymbol{z}}$. The terms proportional to $[\tau_{rec}^{(R)} - \tau_{rec}^{(L)}]$ are not negligible, in general. They describe a slow evolution of the polarization owing to a disparity in the recombination rates in the two wells. Even if a sample were initially prepared with electrons evenly distributed between the wells, $\hat{\rho}(0) = \hat{\rho}^{eq}$, it would acquire a slowly varying transient dipole moments due to these terms—because they mix ρ_0 and ρ_x. Such effects are important in the design of a practical oscillator, since they determine the mean polarization value in a steady-state operation, but they need not concern us in this work—especially because their realistic treatment must involve a model for the dynamics of holes in the double well system. Accordingly, we shall assume $\tau_{rec}^{(L)} \approx \tau_{rec}^{(R)}$ and drop terms proportional to $[\tau_{rec}^{(R)} - \tau_{rec}^{(L)}]$. The remaining relaxation term in (B1) has the effect of slightly renormalizing the value of T_2 (making it anisotropic in the x, y plane). With our assumption [(9)], this term is negligible.

By substituting in (11) the interaction representation of the density matrix:

$$\hat{\rho} = e^{-i\hat{H}_0 t/\hbar}\hat{\rho}^{int}e^{i\hat{H}_0 t/\hbar}, \tag{B3}$$

we transform the Bloch equation (B1) into a frame rotating with the angular velocity ω. The interaction representation of operators

$$\hat{\sigma}_x^{\text{int}} \equiv e^{i\hat{H}_0 t/\hbar}\hat{\sigma}_x e^{-i\hat{H}_0 t/\hbar} = \cos(\omega t)\hat{\sigma}_x - \sin(\omega t)\hat{\sigma}_y \quad \text{(B4a)}$$

$$\hat{\sigma}_y^{\text{int}} \equiv e^{i\hat{H}_0 t/\hbar}\hat{\sigma}_y e^{-i\hat{H}_0 t/\hbar} = \cos(\omega t)\hat{\sigma}_y + \sin(\omega t)\hat{\sigma}_x \quad \text{(B4b)}$$

$$\hat{\sigma}_z^{\text{int}} = e^{i\hat{H}_0 t/\hbar}\hat{\sigma}_z e^{-i\hat{H}_0 t/\hbar} = \hat{\sigma}_z \quad \text{(B4c)}$$

corresponds to rotating the orts:

$$\overset{o}{x} = \cos(\omega t)\overset{t}{x} - \sin(\omega t)\overset{t}{y} \quad \text{(B5a)}$$

$$\overset{o}{y} = \cos(\omega t)\overset{t}{x} + \sin(\omega t)\overset{t}{x}. \quad \text{(B5b)}$$

Consequently, the matrix elements of $\hat{\rho}^{\text{int}}$ satisfy the following set of equations:

$$\frac{d\rho_0}{dt} = -\frac{\rho_0}{\tau_{\text{rec}}} + G(t)$$

$$\frac{d\rho_x^{\text{int}}}{dt} = -\frac{\rho_x^{\text{int}}}{T_2} + G(t)\cos(\omega t)$$

$$\frac{d\rho_y^{\text{int}}}{dt} = -\frac{\rho_y^{\text{int}}}{T_2} - G(t)\sin(\omega t)$$

$$\frac{d\rho_z}{dt} = -\frac{\rho_z - \rho_z^{\text{eq}}}{T_1}, \qquad \text{where } \rho_z^{\text{eq}}(t) = \rho_0(t)\tanh(\hbar\omega/2kT). \quad \text{(B6d)}$$

With the initial condition $n(t) = 0$ for $t < 0$ and, therefore, $\hat{\rho}(0) = \hat{\rho}^{\text{int}}(0) = 0$, these equations have the following solution:

$$\rho_0(t) \equiv n(t) = \int_0^t G(t-t')e^{-t'/\tau_{\text{rec}}}\,dt' \quad \text{(B7a)}$$

$$\rho_x^{\text{int}}(t) = e^{-t/T_2}\int_0^t G(t')\cos(\omega t')e^{t'/T_2}\,dt'$$

$$= \int_0^t G(t-t')\cos\left[\omega(t-t')\right]e^{-t'/T_2}\,dt' \quad \text{(B7b)}$$

$$\rho_y^{\text{int}}(t) = e^{-t/T_2}\int_0^t G(t')\sin(\omega t')e^{t'/T_2}\,dt'$$

$$= \int_0^t G(t-t')\sin\left[\omega(t-t')\right]e^{-t'/T_2}\,dt' \quad \text{(B7c)}$$

$$\rho_z(t) = \frac{\tanh(\hbar\omega/2kT)}{T_1}\int_0^t n(t-t')e^{-t'/T_1}\,dt'$$

$$\approx n(t)\tanh(\hbar\omega/2kT)\,(1 - e^{-t/T_1}). \quad \text{(B7d)}$$

Transforming (B7b) and (B7c) back into the stationary frame finally gives

$$\rho_x(t) = \int_0^t G(t-t')\cos(\omega t')e^{-t'/T_2}\,dt' \quad \text{(B8a)}$$

$$\rho_y(t) = \int_0^t G(t-t')\sin(\omega t')e^{-t'/T_2}\,dt'. \quad \text{(B8b)}$$

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[8] We are not interested here in an accurate model of the recombination kinetics. If the number of photogenerated holes exceeds their background concentration provided by the doping, then the rates $\tau_{\text{rec}}^{(L)}$ and $\tau_{\text{rec}}^{(R)}$ should be thought of as some effective rates which, in a steady state arising from a constant or periodic excitation, depend on the average value of n.

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