COHERENT VERSUS INCOHERENT RESONANT TUNNELING 
AND IMPLICATIONS FOR FAST DEVICES

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Physics of resonant tunneling (RT) in quantum-well structures is reviewed, emphasizing the difference between the truly coherent tunneling, analogous to the resonant transmission through a Fabri-Perot étalon in optics, and the sequential processes, in which the phase of electron wave function is destroyed between two tunneling steps. Several proposals and experimental demonstrations of three-terminal RT configurations are also discussed. In addition to a negative differential resistance in their output circuit, most RT transistors exhibit a negative transconductance, a feature which can lead to the implementation of various high-speed functional logic devices.

1. Introduction

Resonant tunneling (RT) in double-barrier (DB) quantum-well (QW) structures had been originally proposed and discussed as an electron wave phenomenon analogous to the resonant transmission of light through a Fabri-Perot étalon. A discussion of the historical development of these ideas and references to the early work can be found in my recent review.\(^1\) Considering an electron at energy \(E\) incident on a one-dimensional DBQW structure (Fig. 1), one finds that when \(E\) matches one of the energy levels \(E_i\) in the QW, then the amplitude of the electron de Broglie waves in the QW builds up due to multiple scattering and the waves leaking in both directions cancel the reflected waves and enhance the transmitted ones. Near the resonance one has

\[
T(E) = \frac{4 T_1 T_2}{(T_1 + T_2)^2} \frac{\gamma^2}{(E - E_i)^2 + \gamma^2}
\] (1)

where \(T_1\) and \(T_2\) are the transmission coefficients of the two barriers at the energy \(E = E_i\) and \(\gamma \equiv \hbar/\tau\) is the lifetime width of the resonant state [quasi-classically, \(\gamma \approx E_i (T_1 + T_2)\)]. In the absence of scattering, a system of two identical barriers \((T_1 = T_2)\) is completely transparent for electrons entering at resonant energies, and for different barriers the peak transmission is proportional to the ratio \(T_{\text{min}}/T_{\text{max}}\), where \(T_{\text{min}}\) and \(T_{\text{max}}\) are respectively the smallest and the largest of the coefficients \(T_1\) and \(T_2\). The total transmission coefficient, plotted against the incident energy has a number of sharp peaks, as shown in Fig. 1. For a one-dimensional system, the connection between the transmission coefficient and the electrical resistance \(R\) of the DBQW system clad by two electron reservoirs at different chemical potentials, maintained by an external bias, is established by the well-known Landauer formula, \(R^{-1} = (e^2/\hbar) T(E_F)\), which can also be extended to the three-dimensional case via its multi-channel generalizations. A lucid discussion of this approach to RT can be found in the recent paper\(^2\) by Büttiker.

Several years ago, I had argued\(^3\) that the experimentally observed negative differential resistance (NDR) in DBQW diodes can be understood without invoking a coherent Fabry-Perot transmission resonance — but rather as a two-step process in which electrons first tunnel from the emitter electrode into the quasi-bound state in the QW, and then from the well into the collecting electrode. Between these two steps the electron phase memory may be completely lost. For a detailed discussion of the sequential mechanism of operation of RT diodes the reader is referred to the review.\(^1\)

In three-dimensional DBQW diodes, the NDR arises solely as a consequence of the dimensional confinement of states in a QW, and the conservation of energy and lateral momentum in...
Here $n(E)$ is the distribution of incoming particles with respect to the kinetic energy of their motion perpendicular to the barriers; in three-dimensional DBQW structures $n(E_i)$ is proportional to the area of the shaded disk in Fig. 1 of ref. 3. Since it is $T_{\text{min}}$ and not $T$ which determines the RT current, it is clear that the coherent mechanism is not sensitive to the barrier asymmetry. Of course, the situation would be quite different if one were able to study the RT characteristics with quasi-monoenergetic distribution of incoming electrons ($E_F < \gamma$), or if one could design samples in which the resonance would occur at low applied biases such that $e(\mu_1 - \mu_2) < \gamma$. In the latter case the coherent resistance would be again sensitive to the total transmission coefficient, as given by the Landauer formula (such a situation was considered by Büttiker.) However, as far as I know, none of the DBQW structures studied to-date conforms to these specifications.

As discussed by Weil and Vinter, Eq. (2) also describes the tunneling current in the “sequential picture” under similar approximations. Of course, in the absence of scattering there is no “incoherent” tunneling and the sequential picture is rather meaningless. However, Weil and Vinter have argued that Eq. (2) remains valid to first order even in the presence of scattering, provided the energy distribution for incoming electrons is broader than the scattering-limited level width. A legitimate question may then be asked, whether or not there is a meaningful reason to distinguish between the two pictures?

I believe there are at least two reasons for doing so. Firstly, the question of coherent versus incoherent electron transport transcends in importance the mere analysis of static $I$-$V$ characteristics in DBQW diodes. Distinction between the two processes depends on the relative value of the phase-relaxation time $\tau_\phi$ and the tunneling time $\tau_0$. In the instance of resonant tunneling, neither of these two quantities is presently free from ambiguities. In the next section, I shall discuss the processes which lead to phase relaxation; in particular, it will be shown that these are not only inelastic scattering processes. Quantitatively, the effect of scattering on the $I$-$V$ characteristics is rather poorly understood at this time. As will be discussed in Sect. 2, the peak-to-valley ratio in RT current can be strongly affected by processes which mix the longitudinal and the transverse components of the electron wave function. Secondly, the sequential-tunneling approach provides a natural framework for discussing a new class of three-terminal RT devices, especially those which essentially rely on the NDR property of tunneling into a QW.
without an attendant (coherent or incoherent) second tunneling step. It is my opinion that the most important future applications of RT are associated with multi-terminal devices, because of their potential for an enhanced functionality "per terminal" in integrated circuits. Some of the recent proposals and experimental demonstrations of RT transistors with a negative transconductance will be discussed in Sect. 3.

2. Effects of Electron Scattering

Consider a single electron incident on a DBQW structure, assumed for simplicity to have $T_1 = T_2$. The reflected-wave amplitude represents the sum of amplitudes of all quantum-mechanical paths corresponding to multiple reflections from the barriers; at resonance the phases of different amplitudes combine so as to cancel the net reflected wave. If some of the paths contain an external interaction vertex, which changes the wave-function phase by a random amount of order $\pi$, the reflected wave will not be canceled. It is quite unimportant, whether the phase-randomizing interaction is inelastic or not. For example, in a one-dimensional case one can think of a magnetic impurity which flips the electron spin without changing the energy; clearly, partial waves of opposite spin do not cancel each other. For a three-dimensional DBQW structure an elastic scattering event may change the direction of electron momentum in the $xy$-plane (the plane of the barriers); although the factor describing the electron wave-function in the tunneling direction has not changed, the overall phase has, and no cancellation is possible. In a Gedanken experiment measuring the single-electron transmission coefficient the relevant phase relaxation time $\tau_\phi$ is, therefore, at least as short as the momentum relaxation time $\tau_m$ in the QW, as determined by mobility measurements. In fact, one can even have $\tau_\phi \ll \tau_m$, since obviously $\tau_m$ gets no contribution from the electron-electron (ee) scattering, which is just as important as the impurity and the phonon scattering for altering the single-electron phase.

It should be clearly understood, however, that the phase memory is not necessarily lost in an elastic scattering event, so that another such event can restore the single-electron phase. As is well known, the interference of scattered waves leads to quantum corrections to the metallic conductivity measured in experiments on a mesoscopic scale. In such experiments, the relevant $\tau_\phi$ is determined by the irreversible phase degradation brought about by the electron interaction with an equilibrium reservoir of scatterers. Although this time is sometimes loosely thought of as the inelastic scattering time $\tau_{\text{in}}$, strictly speaking this is not so: an irreversible phase degradation can be also produced by an interaction with a degenerate level of the reservoir. In a double-slit experiment, no interference is possible if one of the interfering paths involves interactions with an external system — leaving that system in a state orthogonal to its initial state. It is thus clear that the question of what is the relevant phase relaxation time can be decided only with respect to a specific experiment in mind. As discussed in the Introduction, static I-V curves of a DBQW diode are rather insensitive to this question.

Even if one could design a DBQW structure in which the energy distribution of incoming electrons were arbitrarily narrow, one would still observe only an inhomogeneous average of the transmission coefficient $T$ over the device area. Only for mesoscopic devices with transverse dimensions comparable to the phase coherence length $(D\tau_\phi^{\text{int}})^{1/2}$ ($D$ being the diffusion coefficient), the static I-V curves can be expected to exhibit interference effects associated with the phase memory retention over the time $\tau_\phi^{\text{int}}$; to my knowledge, however, all studied DBQW diodes are macroscopic and these effects are washed out.

With a model estimate for $\tau_\phi$, one can determine whether the RT is dominated by coherent or incoherent processes by comparing $\tau_\phi$ with the "tunneling time" $\tau_0$, which should be understood as the lifetime of the resonant state limited by its decay due to tunneling. Quasiclassically, the ratio $\tau_0/\tau_\phi$ corresponds to the average number of bounces an electron makes inside the QW relative to the average number of phase randomizing events it is expected to face while bouncing back and forth. As discussed by many authors, $2,3,1$ $\tau_0$ is the time which limits the oscillation frequency of DBQW diodes.

Clearly, in any experiment the relevant $\tau_\phi \ll \tau_\phi^{\text{int}}$. It would be instructive to discuss this issue in the instance of the recently proposed time-resolved luminescence experiment capable of a direct observation of the time evolution in heterostructure barrier tunneling. Consider an idealized structure, Fig. 2, containing two QW's separated by a barrier; the wells have identical ground-state levels $(E_1 = E_2 \equiv E_0)$ in the conduction band — but different in the valence band. This allows a selective preparation of an initial electron state by an ultra-short interband photo-excitation. In a coupled QW system electrons will oscillate between the two wells, giving rise to an oscillating luminescence signal with a period directly related to the tunneling time. As is well known, in the presence of a tunnel coupling, the single-well states $|1\rangle$ and $|2\rangle$ are not stationary. If, immediately upon the excitation, electrons are
local configuration of elastic scatterers. An equivalent way of describing this situation is to add a time dependent phase $\phi_j(t)$ in the argument of the trigonometric functions in (3) -- different for each channel $j$. The relative phases of different elastic channels will therefore randomize and upon the time $\tau_\phi$ no luminescence oscillation will be observed. One can even assume that the elastic processes involved do conserve the phase memory ($\tau_\phi^{\text{tr}} = \infty$), which means only that the functions $\phi_j(t)$ are perfectly deterministic. In principle, this would leave an experimenter with the possibility of doing a "time-reversal" trick similar to the well-known spin-echo experiments in nuclear magnetic resonance. The times $\tau_\phi^{\text{tr}}$ and $\tau_\phi$ can be viewed as analogs of the longitudinal ($t_1$) and the transverse ($t_2$) relaxation times, respectively. If after a time $\Delta t < \tau_\phi^{\text{tr}}$ the momenta of all electrons in the double-well system could be reversed simultaneously, then after another $\Delta t$ the luminescence oscillations will re-emerge to last for another period of order $\tau_\phi$; as far as I am aware, nobody knows how to do such an experiment at this time.

Let us turn our attention to another issue, associated with the effect of scattering on resonant tunneling: namely the mixing of ancillary degrees of freedom (those corresponding to the electron motion in $xy$ plane) with the longitudinal component of the wave function. This problem has been discussed with exceptional clarity by Meshkov. He considered the wave function of electrons confined to a QW, bounded by an infinite barrier on one side and a finite-height barrier $V(z)$ on the other. In the absence of ee interaction and inhomogeneities, the free motion in $xy$ plane is completely separable from the quantized longitudinal motion. Consequently, the wave function decays into the bulk with the characteristic exponential

$$\Psi \propto \exp \left( -\frac{\hbar}{\beta} \int \sqrt{2m[V(z) - E_0]} \, dz \right),$$

where $E_0$ is the energy of the subband bottom in the QW. The tunneling exponent (4) is independent of the kinetic energy $K$ of the electron motion in $xy$ plane. The situation is qualitatively different in the presence of scattering -- which mixes different degrees of freedom. However weak the scattering processes, the asymptotic decay law for the electron density is described by a wavefunction that would result if the carriers had tunneled in the one-dimensional potential $V(z)$ -- but with the total energy $E = E_0 + K$:

$$|\Psi|^2 \propto \exp \left( -2\frac{\hbar}{\beta} \int \sqrt{2m[V(z) - E]} \, dz \right).$$
Meshkov has rigorously proven this statement in a quite general form. At a sufficiently large distance from the QW the decay rate (5) is strictly valid; transition to the no-scattering limit (4) is described by a pre-exponential factor, which depends on the specific scattering mechanism and which has been evaluated for several model examples, including the short-range ee interaction and the scattering by inhomogeneities of the structure.

Although these considerations have not been applied to the case of tunneling in DBQW structures, it is my opinion that similar effects should have an important role there too. In particular, one can expect a strong effect on the peak-to-valley ratio in current. Experiments to-date seem to also support this proposition, as discussed recently by Wolak et al. It also gives a natural explanation to the fact, first noted by Shewchuk et al., that highest peak-to-valley ratios are obtained in DBQW diodes with lightly doped or undoped regions inserted immediately outside the barriers. Another consequence of the mixing of longitudinal and transverse motions by elastic scattering, which is worth investigating, is the dependence of the lifetime of a resonant state on its kinetic energy, \( \tau_0 \propto \tau_0 \). In particular, in time-resolved luminescence experiments with a single QW, one can expect higher tunneling escape rates when the quasi-Fermi level of QW electrons is increasing. In heterojunction superlattices, the mixing of different degrees of freedom may lead to a Fermi-level dependence of the subband effective mass.

It is reasonable to conclude that any quantitative discussion of the RT in DBQW diodes must be based on a concrete model of scattering processes appropriate for an experimental structure under consideration.

3. Resonant-Tunneling Transistors

Many workers have appreciated the attractive possibilities which would arise from an integration of the double-barrier RT structure in a three-terminal device. References to various proposals in this regard can be found in the reviews. Below, we shall discuss two unipolar transistors, based on resonant tunneling, whose characteristics \( l_{out}(V_{in}) \) possess regions of both positive and negative transconductance. This is an important property, because it allows the implementation of novel functional circuit configurations, analogous to those available in the celebrated complementary silicon (CMOS) technology. Indeed, the main advantage of CMOS circuits results from the fact that transconductances of p- and n-channel transistors are of opposite sign, which allows high-speed switching combined with a low power dissipation in the steady state. Similar circuits can be obtained from unipolar RT transistors: a pair of such transistors, one operating in the positive the other in the negative transconductance range, is electrically equivalent to the CMOS inverter and can perform its logic functions at low power dissipation.

3.1 Quantum Wire Transistor. This device, proposed by Luryi and Capasso and illustrated in Fig. 3, uses a linear rather than planar QW ("quantum wire") as the active region. Electrons resonantly tunnel from a 2D emitter into (or through) 1D QW states; potential difference between the QW and the emitter, and therefore the RT current, can be controlled electrostatically with an external gate. The 1D confinement can be achieved with the help of a V-groove etch of a planar DBQW structure followed by an epitaxial overgrowth with gate layers, or a similar processing of a vertical <110> edge.

Application of a positive gate voltage \( V_G \) induces 2-D electron gases at the two interfaces with the edges of undoped layers outside the QW. These gases will act as the source (S) and drain (D) electrodes. The bottom of the 2-D subband \( E_0 \) is split up from the classical conduction band edge \( E_C \) by the dimensional confinement in \( y \) direction. At
The transconductance characteristic $I_D(V_G)$ near the RT peak can be obtained by scaling the source-to-drain diode curve by the gate leverage factor $\lambda$, given by

$$\lambda \equiv \frac{\partial I_D / \partial V_G}{\partial I_D / \partial V_D} = -\left(\frac{\partial V_D}{\partial V_G}\right)_{I_D=\text{const}} \frac{\Psi_z(QW)}{\Psi_b(QW)},$$

where the coordinates of the QW are substituted in the arguments of the right-hand side. In the example illustrated in Fig. 4, the total source-to-drain gap is 5 times the gate thickness, resulting in $\lambda \approx 4.6$, which means that the gate in this example is more effective than the drain.

3.2 Gated Quantum-Well RT Transistors. Recently, Beltram et al.\textsuperscript{20} demonstrated a three-terminal RT device, in which the QW was used as a collector, separated by a thin tunneling barrier from a doped emitter layer on one side and bounded by an insulated gate on the other side. The band diagram of the device structure under bias is schematically illustrated in Fig. 5. Electrons tunnel into the second (empty) subband of the QW, while the highly conducting 2D electron gas (2DEG) in the ground subband permits application of an external bias to the QW. Electron transport from the emitter to collector thus proceeds in two steps:
tunneling through a single barrier and a subsequent drift laterally. Obviously, the sequential tunneling picture can be naturally extended to describe such a transport and one expects an NDR in the collector circuit at a fixed gate bias.\textsuperscript{21}

Transistor action is achieved by the modulation of the position of the 2-dimensional subbands in the QW with respect to the emitter Fermi level by the electric field emanating from the gate electrode. This occurs for two distinct reasons. One, which in accordance with Bonnefoi et al.\textsuperscript{22} can be termed a generalized Stark effect, is associated with the gate field penetration into the QW and the sensitivity of the QW energy levels to the shape of the well. The other effect is the quantum capacitance\textsuperscript{23} of a 2DEG, as a result of which the gate field partially penetrates beyond the QW collector and induces charges on the emitter electrode.

Beltram et al.\textsuperscript{20} have observed the negative transconductance effect (predicted for such a structure in ref. 1, p. 556) as well as the NDR effect at a fixed gate bias.\textsuperscript{21} A good quantitative agreement was found between the measured and the calculated characteristics. The effectiveness of the gate in controlling the tunneling current can be described by a gate leverage factor [cf. Eq. (7)], here conveniently defined as follows:

$$\lambda \equiv \frac{\delta E_1}{e \delta V_G} = \lambda_S + \lambda_Q,$$

where $\lambda_S$ and $\lambda_Q$ are, respectively, the contributions of the Stark effect and the quantum capacitance. The former can be evaluated in first-order perturbation theory by taking the expectation value

$$\delta E_1 \approx \left< \psi_1 | \delta \phi(z) | \psi_1 \right>$$

of the electrostatic potential variation in the QW (calculated self-consistently with the ground-subband wavefunction $\psi_0$) over the unperturbed upper-subband wavefunction $\psi_1$. The quantum-capacitance contribution is given by

$$\lambda_Q = \frac{C_1}{C_1 + C_2 + C_Q},$$

where $C_1$ and $C_2$ are the geometric gate-collector and emitter-collector capacitances, respectively, and

$$C_Q = \frac{m_e^2}{\pi \hbar^2}$$

is the quantum capacitance of the 2DEG, which is a characteristic of the QW material only ($C_Q \approx 4.5 \mu \text{m/cm}^2$ for GaAs). Thus calculated total $\lambda$ agrees with the experimentally measured gate leverage factor\textsuperscript{20} to better than 10%.

### 4. Conclusion

Physics of resonant tunneling in double-barrier quantum-well structures has been reviewed, emphasizing the difference between the truly coherent tunneling, analogous to the resonant transmission through a Fabri-Pérot étalon in optics, and the sequential processes, in which the phase of electron wave function is destroyed between two tunneling steps. Although the two mechanisms are undoubtedly distinct and correspond to radically different single-electron transmission coefficients, in most experimentally studied DBQW diodes they lead to practically indistinguishable current-voltage characteristics. The salient – and the most useful – feature of these characteristics, namely the two-terminal negative differential resistance, in both pictures results from the reduced dimensionality of electronic states in the QW and the conservation of parallel momentum in tunneling. The NDR of the RT diodes has its main potential application in fast oscillators; with respect to such devices, the question of coherent versus sequential mechanism may affect the theoretical limit frequency. A wider range of potential applications for RT in QW structures is associated with multiterminal transistor-like configurations. In addition to the NDR in their output circuit, most RT transistors exhibit a negative transconductance, a feature which can lead to the implementation of various high-speed functional logic devices. Some of the RT
transistors can be discussed equally well within both the coherent and the sequential pictures, for others the latter picture is better suited, as it allows one to associate the NDR exclusively with the first tunneling step, leaving means of electron extraction from the QW at the designer's disposal.

Acknowledgement — I wish to thank R. K. Smith for computer-generating the electrostatic-potential curves in Figure 4 and for a helpful discussion.

References


6. A similar argument had been advanced by M. C. Payne, J. Phys. C 19, 1145 (1986).

7. Recently Rita Gupta and B. K. Ridley [J. Appl. Phys., to be published] have shown that scattering reduces the peak-to-valley ratio of the NDR curves in DBQW diodes in a non-equivalent way for the coherent and the incoherent mechanisms, the degradation effect being larger for the coherent process. Thus, a systematic study of the static IV curves with a controlled variation in the strength of scattering may, in principle, distinguish between the two mechanisms.

8. A lucid discussion of this point was presented by Yakir Aharonov at the Workshop on Novel Phenomena in Mesoscopic Physics, Rehovot, Israel (July, 1988).


18. V-groove quantum wires have been first considered by H. Sakaki, Jpn. J. Appl. Phys. 19, L735 (1980), who suggested the possibility of achieving an enhanced mobility along the wire due to suppressed scattering.


