Mid-Term Test 1: ESE 558 Digital Image Processing

Date: 3/21/06, Duration: 2 hours 15 mins, Spring 2006, Max. Marks: 35 points

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This test is open text books. Show all steps to demonstrate your understanding and to get full credit.

1. (8=3+3+2 pts)

   (a) (3 points) Explain how certain characteristics of the human eye provide it a large dynamic range for brightness sensing (e.g. the eye can see under very low illumination such as moon light and also very bright illumination such as sun light, but not simultaneously).

   (b) (3 points) A digital camera has focal length f mm and pixel size p mm. Derive an expression for the spatial resolution (lines/mm) of the camera as a function of object distance z. Assume that the distance between the lens and the CCD image plane is a constant equal to the focal length f.

   (c) (2 point) If $f = 20$ mm and $p = 0.010$ mm for a camera, what is its spatial resolution at a distance of 500 mm?

2. (7 = 1+1+2+3 pts) A 3 bits/pixel image of size 5x5 is given here. At the center pixel (2,2), find the following:

```
                   IMAGE f(x,y)
y     x=|  0    1    2    3    4    |
||      |                           |
-----------------------------------    Laplacian      Low-pass filter
0       |  4    7    6    2    1    |
        |                           |  0   1   0      0.02   0.10  0.02
1       |  2    4    6    1    0    |
        |                           |  1  -4   1      0.10   0.52  0.10
2       |  4    4    7    5    4    |
        |                           |  0   1   0      0.02   0.10  0.02
3       |  3    0    3    2    1    |
        |                           |
4       |  5    7    5    1    6    |
        |                           |
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   (a) (1 pt) The output of a 3x3 median filter at (2,2).

   (b) (1 pt) The output of the 3x3 Low-pass filter shown above at (2,2).

   (c) (2 pt) The result of unsharp-masking at (2,2) (assume suitable parameters if needed). Show your steps. Use the Laplacian filter shown above.

   (d) (3 pts) The result of histogram equalization at the point (2,2). Show steps in obtaining your solution.

3. (4 points) Prove the *convolution property* of the one-dimensional continuous Fourier transform. Show all steps. In particular, do not use other properties of the Fourier transform to skip steps.

4. *Sampling theorem:* (5 points)
   An image is specified by $f(\mathbf{r}) = \cos 2\pi(\mathbf{w_0} \cdot \mathbf{r})$ where $\mathbf{r} = (x, y)$ and $\mathbf{w_0} = (-0.2, 0.7)$. The image is sampled on a lattice $\mathbf{r_{mn}} = m\mathbf{r_1} + n\mathbf{r_2}$ where $\mathbf{r_1} = (1, 0)$, and $\mathbf{r_2} = (0, 1)$ for $m, n = 0, \pm1, \pm2, \pm3, \cdots$.. The reciprocal lattice is given to be $\mathbf{w_{mn}} = m\mathbf{w_1} + n\mathbf{w_2}$ where $\mathbf{w_1} = (1, 0)$ and $\mathbf{w_2} = (0, 1)$.

   If $G(\mathbf{w}) = \text{circ}(\mathbf{w})$ (where $\mathbf{w} = (u, v)$) is the Fourier transform of the interpolation filter $g(\mathbf{r})$ used in reconstructing the image from its samples, find an explicit expression for the reconstructed image $f'(\mathbf{r})$. The function $\text{circ}(\mathbf{w})$ has a value of 1.0 for $|\mathbf{w}| \leq 0.5$ and 0.0 elsewhere. That is, it has as value of 1 inside a circle centered at the origin and of radius 0.5, and zero outside.

5. *Optimal quantization: (4 points)*
   The probability density function of an image signal z is

$$p(z) = \frac{2}{3}\left(1 - |z|^3\right) \quad \text{for } |z| \leq 1, \quad \text{and} \tag{1}$$
$$= 0 \quad \text{for } |z| > 1. \tag{2}$$

   We need to optimally quantize the signal to two distinct levels q_1, q_2 with a decision level z_2. Guess a solution for z_2 using common sense and then solve for q_1, q_2.

6. (5+2=7 points) Image Sampling using Orthonormal Functions
   A square integrable continuous picture function $f(x, y)$ can be expanded in terms of a set of orthonormal basis functions $\phi_{mn}(x, y)$ for $m, n = 0, 1, 2, \cdots$, and the coefficients a_{mn} of the expansion can be taken as picture samples.

   (a) Show that over the rectangle defined by $-A/2 \leq x \leq A/2$, $-B/2 \leq y \leq B/2$, the following functions form an orthonormal set (no need to prove completeness).

$$\phi_{mn}(x, y) = \frac{1}{\sqrt{AB}}\, exp\left[j2\pi\left(\frac{mx}{A} + \frac{ny}{B}\right)\right], \quad \text{for } m, n = 0, 1, 2, 3, \cdots. \tag{3}$$

   (b) How would you pick a finite number, say 100, of $\phi_{mn}(x, y)$ to accomplish optimal sampling for a given picture function?