Mid-Term Examination: ESE 558 Digital Image Processing

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Show all steps to demonstrate your understanding and get full credit.

Part I: Closed-book. Time limit: 1 hour.

1. Optimal quantization: 5 points.
   (a)Formulate and solve the optimal gray-level quantization problem discussed in class. You need to derive equations for optimal quantization levels and threshold levels.
   (b) For the equations that you derived, describe a numerical algorithm for computing the optimal parameters for an arbitrary probability density function of the image gray-level.

2. Project: 5 points.
   A gray-level (black/white) image is stored in an array `float f[M][N]`. A one dimensional Gaussian filter is stored in `float h[7]`. Wrte a C/C++ function for *separable circular discrete convolution* of f and h to compute the output image `float g[M][N]`. Assume that all array memories have been allocated and initialized. You just need to compute g. There is no need to read or write any data.

   Part II: Open-book (clean text book only; no other materials).

3. (a) (2 points) A digital camera has focal length f mm and pixel size p mm. Derive an expression for the spatial resolution (lines/mm) of the camera as a function of object distance z. Assume that the distance between the lens and the CCD image plane is a constant equal to the focal length f.
   (b) (1 point) If $f = 20$ mm and $p = 0.010$ mm for a camera, what is its spatial resolution at a distance of 2 meters?
   (c)(2 points) The ratio of maximum to minimum image brightness perceived by the human eye at any time instant is very limited. However, we can still see under very low and very high illumination levels (e.g. full moon light, and mid-day sunlight). Explain precisely and briefly how this is accomplished by the human eye.

4. (a)(2 points) Describe the advantages/disadvantages of mean and median filters for image smoothing. Consider all practical issues.
   (b) (2 points) Derive a discrete convolution filter for estimating $\frac{\partial^2 f(x,y)}{\partial x \partial y}$.
   (c)(2 points) Compare the relative advantages/disadvantages of implementing a filter in (i) the spatial domain as a discrete convolution, versus (ii) the Fourier domain as a multiplication of two DFTs.

5. (5 points) The Fourier transform of $f(x, y)$ is given to be $F(u, v)$. Starting from the definition of the continuous Fourier transform, derive the Fourier transform of $f(a(x - b), c(y - d))$ in terms of $F(u, v)$ where a, b, c, d are scalar constants.

6. (1+3 points) An image is specified by $f(\mathbf{r}) = \cos 2\pi(\mathbf{w_0} \cdot \mathbf{r})$ where $\mathbf{r} = (x, y)$ and $\mathbf{w_0} = (0.2, 0.6)$. The image is sampled on a lattice $\mathbf{r_{mn}} = m\mathbf{r_1} + n\mathbf{r_2}$ where $\mathbf{r_1} = (1, 0)$, and $\mathbf{r_2} = (0, 1)$ for $m, n = 0, \pm 1, \pm 2, \pm 3, \cdots$..
(i) Find the reciprocal lattice $\mathbf{w_{mn}} = m\mathbf{w_1} + n\mathbf{w_2}$ (i.e. find $\mathbf{w_1}$ and $\mathbf{w_2}$).
(ii)If $G(\mathbf{w}) = \mathrm{rect}(\mathbf{w})$ is the Fourier transform of the interpolation filter $g(\mathbf{r})$ used in reconstructing the image from its samples, find an explicit expression for the reconstructed image $f'(\mathbf{r})$.