Mid-Term Examination: ESE 558 Digital Image Processing

Date: 3/25/2003, Duration: 2 hours 30 mins, Spring 2003

SUNY at Stony Brook, Murali Subbarao, Max. Marks: 30

In answering the following questions, write all steps to demonstrate your understanding.

PART I: Basic facts and project (Closed book)

1. (a) (1 point) Define the 2D DFT of a digital image f[m][n] of size MxN.

   (b) (1 point) Define the 2D convolution of a digital image f[m][n] of size MxN with a digital filter h[m][n] of the same size.

   (c) (4 points) For a continuous image f(x,y) with non-zero values in a rectangular region, (i) define the ortho-normal expansion of f(x,y) interms of a set of ortho-normal basis functions $\phi_{mn}(x, y)$. (ii) Specify the properties to be satisfied by $\phi_{mn}(x, y)$, and give an expression for the (iii) ortho-normal expansion and (iv) for the coefficients of the expansion.

   (d) (4 points: Project) Given a digital image stored in a floating point array f[M][N], with fixed array size MxN, write a computer program in a high level language (e.g. C/C++) to compute and print the Discrete Fourier Transform of f[m][n]. You can assume that the image data has already been read into f[m][n], and memory has been allocated for all arrays that you need (i.e. there is no need to write dynamic memory allocation function). You should compute F[u][v] so that origin should be in the center. Also, you should compute using the separability property (first along rows, then along columns). The output is simply the floating point numbers representing the Fourier coefficients (real and imaginary parts). There is no need to deal with issues of data read/write or compress/decompress. Just consider raw data.

PART II: Open book)

2. (a) (2 points) When you enter a dark theater on a bright day, it takes an appreciable interval of time before you can see well enough to find an empty seat. Which of the computational processes of the human visual system explained in the Gonzales and Woods text book are at play in this situation and how?

   (b) (3 points) A CCD camera chip of dimensions 10x10 mm, and having 1000x1000 pixel elements is focused on a square, flat area, located 500 mm away. The focal length of the camera is 20 mm. How many line pairs per mm will this camera be able to resolve?

3. (5 points)*Spatial-domain filtering/convolution of digital images*

   A 5×5 discrete mean filter h is separable along the row and column directions. If $\star$ denotes convolution, we can denote this by h = hr $\star$ hc where

```
                           hr * hc = h

     | 0 0 0 0 0 |         | 0 0 1 0 0 |            | 1 1 1 1 1 |
   1 | 0 0 0 0 0 |       1 | 0 0 1 0 0 |      1     | 1 1 1 1 1 |
hr = - | 1 1 1 1 1 |  hc = - | 0 0 1 0 0 |  h = --- | 1 1 1 1 1 |
   5 | 0 0 0 0 0 |       5 | 0 0 1 0 0 |     25     | 1 1 1 1 1 |
     | 0 0 0 0 0 |         | 0 0 1 0 0 |            | 1 1 1 1 1 |
```

Let f[m][n] be a digital image of size $M \times N$.

(a) Describe in a few sentences a computationally efficient algorithm for implementing f $\star$ hr. (Hint: a straight-forward implementation would involve four additions and one multiplication operation per pixel, but your algorithm should require lesser computation per pixel on average).

(b) Describe in a few sentences a computationally efficient algorithm for implementing the two-dimensional convolution f $\star$ h as two one-dimensional convolutions as f $\star$ h = (f $\star$ hr) $\star$ hc.

(c) Estimate roughly the computational speed-up of your algorithm in comparison with direct two-dimensional convolution that involves 24 additions and one multiplication per pixel. Assume that any arithmetic operation (add, subtract, multiply) takes the same amount of CPU time to compute.

4. (5 points) Given the DFT F[u][v] of a digital image f[m][n], derive an expression for the discrete partial derivative estimated by f[m+1][n]-f[m][n] in terms of F[u][v]. Show that this partial derivative is equivalent to a sort of high-pass filtering.

5. *(5 points, RK, Ch. 4, Digitization and Aliasing)*
An image is specified by $f(\mathbf{r}) = cos 2\pi(\mathbf{w_0} \cdot \mathbf{r})$ where $\mathbf{r} = (x, y)$ and $\mathbf{w_0} = (0.3, 0.8)$. The image is sampled on a lattice $\mathbf{r_{mn}} = m\mathbf{r_1} + n\mathbf{r_2}$ where $\mathbf{r_1} = (1, 0)$, and $\mathbf{r_2} = (0, 1)$ for $m, n = 0, \pm 1, \pm 2, \pm 3, \cdots$..
(i) Find the reciprocal lattice $\mathbf{w_{mn}} = m\mathbf{w_1} + n\mathbf{w_2}$ (i.e. find $\mathbf{w_1}$ and $\mathbf{w_2}$).
(ii)If $G(\mathbf{w}) = rect(\mathbf{w})$ is the Fourier transform of the interpolation filter $g(\mathbf{r})$ used in reconstructing the image from its samples, find an explicit expression for the reconstructed image $f'(\mathbf{r})$.