

QUESTION # 3 (10 points) :

Image sampling using orthonormal functions.

Let  $f(x, y)$  be a continuous image function defined over a rectangular region  $-A/2 \leq x \leq A/2$ , and  $-B/2 \leq y \leq B/2$  in the  $xy$ -plane. Assume  $f(x, y)$  to be zero outside the rectangular region. Let  $\phi_{mn}(x, y)$  be a set of *orthonormal functions* used for sampling  $f(x, y)$  for  $m, n = 0, 1, 2, \dots$ .

1. (3 points) Define all the conditions that must be satisfied by  $\phi_{mn}(x, y)$  to form an orthonormal set of functions.
2. (2 points) Give an expression for sampling or expanding  $f(x, y)$  in terms of  $\phi_{mn}(x, y)$ .
3. (5 points) Show that, within the rectangular region defined above, the following functions form an orthonormal set:

$$\phi_{mn}(x, y) = \frac{1}{\sqrt{AB}} \exp \left[ j2\pi \left( \frac{mx}{A} + \frac{ny}{B} \right) \right], \quad m, n = 0, 1, 2, \dots \quad (2)$$

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QUESTION # 8 (10 points) :

Restoration of motion blurred images.

A traffic monitoring video camera records a moving car. The motion of the car is parallel to the image plane and produces a continuous translating image pattern  $f(x, y)$  on a uniform (constant brightness) background. The image moves with a uniform speed  $v$  mm/second along the  $x$ -axis on the image plane. The period of exposure for one image frame is  $T$  seconds. During the exposure period of  $-T/2$  to  $T/2$  seconds for an image frame, the camera aperture is fully open with constant area. An image frame of the video is blurred due to the motion of the car.

1. (6 points) Derive the transfer function corresponding to the motion blur for one image frame of the car.
2. (4 points) Assume that the power spectrum of the image noise is modeled by a constant  $C$  and the power spectrum of the image signal is modeled by a Gaussian of the form  $S(u, v) = A \exp - \sigma^2 2\pi(u^2 + v^2)$ . Here  $A$  is some constant,  $\sigma$  is the Gaussian parameter, and  $u, v$  are spatial frequencies.
  - (a) Give an expression for the Wiener (optimal least-square) filter for restoring the motion blurred image frame of the car.
  - (b) Draw a qualitative plot of the MTF of the Wiener filter.

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