PH.D. QUALIFYING EXAMINATION STUDENT'S NAME ____________________

SIGNAL PROCESSING

APRIL 2002

QUESTION #7:

This question has to do with a deterministic (no noise) reconstruction of a two-dimensional (2D) continuous function $f(x, y)$ from its one-dimensional (1D) parallel-beam continuous projections $p_\theta(t)$. Here, θ is the angle of the projection and t the projection coordinate, so that the (t, θ) coordinate system is a rotated version of the (x, y) coordinate system.

1. Write down the Central Slice Theorem. You needn't prove it. Just write it clearly, defining all terms and notation.

2. Write down the formula for reconstructing $f(x, y)$ from its projections via FBP (Filtered Backprojection). No proof needed, but again, a clear definition of terms and symbols is needed.
   Note that FBP may be expressed as an operation in which one first convolves $p_\theta(t)$ with a filter $q(t)$, then backprojects. In this form of FBP, what is $q(t)$?

3. The theme of this question is that 2D image filtering can be accomplished by 1D projection filtering plus backprojection. Let's say that one wants to post-filter $f(x, y)$ with a convolution filter $h(x, y)$ to obtain $g(x, y) = f(x, y) * *h(x, y)$, where ** denotes two-dimensional convolution. [Note that $h(x, y)$ need not be rotationally symmetric.]
   Given the notation and results from parts (1) and (2), show that $g(x, y)$ can be obtained by an FBP method where the projection data $p_\theta(t)$ of $f(x, y)$ are, before backprojection, one-dimensionally convolved with an angularly dependent filter of the form $q(t) * r_\theta(t)$ with * denoting 1D convolution. [Hint: The new filter of course depends on $h(x, y)$.] Write an expression for this new filter.

It is important to be especially clear about your notation and definitions for all parts of the problem.

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PH.D. QUALIFYING EXAMINATION STUDENT'S NAME ____________________

COMMUNICATIONS AND SIGNAL PROCESSING APRIL 2002

QUESTION #8:

1. (5 points) *Symmetric PSF and its OTF*
The Point Spread Function (PSF) of a camera is given by a continuous function $h(x, y)$ of image coordinates x, y. If $h(x, y)$ is real valued (imaginary part is zero) and symmetric (i.e. $h(x, y) = h(-x, -y)$), then prove that the corresponding Optical Transfer Function (OTF which is the Fourier transform of the PSF) is also real valued and symmetric. Your proof should begin from the definition of the two-dimensional continuous Fourier Transform.

2. (5 points) *Spatial-domain filtering/convolution of digital images*
A 5×5 discrete mean filter h is separable along the row and column directions. If $\star$ denotes convolution, we can denote this by h = hr $\star$ hc where

$$\mathrm{hr} * \mathrm{hc} = \mathrm{h}$$

$$\mathrm{hr} = \frac{1}{5}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \mathrm{hc} = \frac{1}{5}\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad \mathrm{hc} = \frac{1}{25}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Let f[m][n] be a digital image of size $M \times N$.

(a) Describe in a few sentences a computationally efficient algorithm for implementing f $\star$ hr. (Hint: a straight-forward implementation would involve four additions and one multiplication operation per pixel, but your algorithm should require lesser computation per pixel on average).

(b) Describe in a few sentences a computationally efficient algorithm for implementing the two-dimensional convolution f $\star$ h as two one-dimensional convolutions as f $\star$ h = (f $\star$ hr) $\star$ hc.

(c) Estimate roughly the computational speed-up of your algorithm in comparison with direct two-dimensional convolution that involves 24 additions and one multiplication per pixel. Assume that any arithmetic operation (add, subtract, multiply) takes the same amount of CPU time to compute.