

A Fast Point-to-Tangent Plane Technique for Multi-view Registration

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Abstract

This paper addresses a registration refinement problem and presents an accurate and fast Point-to-(Tangent) Plane technique. Point-to-Plane approach is known to be very accurate for registration refinement of partial 3D surfaces. However, the computation complexity for finding the intersection point on a destination surface from a source control point is hindering the algorithm from real-time applications. We introduce a novel Point-to-Plane registration technique by combining the high-speed advantage of Point-to-Projection technique. In order to find the intersection point fast and accurately, we forward-project the source point to the destination surface and reproject the projection point to the normal vector of the source point. We show that iterative projections of the projected destination point to the normal vector converge to the intersection point. By assuming the destination surface to be a monotonic function in a new 2D coordinate system, we show contraction mapping properties of our iterative projection technique. Experimental results for several objects are presented for both pair-wise and multi-view registrations.

1. Introduction

Registration refinement of multiple range images is an essential step in multi-view 3D modeling. When the range images are coarsely registered by *a priori* knowledge of registration parameters, registration refines the rigid transformation parameters to minimize alignment error between overlapping range surfaces. There have been many investigations on the refinement problem [7, 17]. For a pair-wise registration, the registration problem can be considered to be an error minimization of the rigid body transformation between control point sets [1] on the pair of surfaces. Control point sets are decided by matching either geometric [5], photometric [12, 18], or geometric and photometric [4, 11] structures of the range surfaces. In this paper, we address a registration problem in the first category, which considers

only geometric structure for control point matching.

Based on the method of control point matching, three approaches can be considered in general. *Point-to-Point*, e.g. Iterative Closest Point (ICP) algorithm [5], *Point-to-(Tangent) Plane* [8, 3], and *Point-to-Projection* [6] techniques are well-known registration techniques. The ICP algorithm is one of the common techniques for refinement of partial 3D surfaces (or models) and many variant techniques have been investigated. However, searching the closest point in the ICP algorithm is a computationally expensive task. In order to accelerate the speed of closest point searching, some searching techniques are commonly employed, for example a kd-tree searching, z-buffering, or a closest-point caching [2, 10, 16].

In contrast, *Point-to-Projection* approach finds the correspondence of a source control point by projecting the source point onto a destination surface from the point of view of the destination [6, 15]. When the source point is backward projected to the image plane of the destination surface, the destination control point is the forward projection of the image point. This approach makes registration very fast, because it does not involve any searching step to find the correspondence. However, one of its disadvantages is that the result of registration is not as accurate as those of the others [17].

Among three approaches, *Point-to-(Tangent) Plane* technique is known to be the most accurate [13, 17]. From a source control point, the matching control point is the projection of the source point onto the tangent plane at a destination surface point which is the intersection of the normal vector of the source point [3, 8, 9]. However, finding the intersection on the destination surface is also computationally expensive. One of the acceleration techniques is first searching the closest point, and finding the intersecting surface (or the triangle) from its neighboring triangles [17]. Gagnon [9] tries to find the intersection on a 2D grid of range images along the projection of the normal vector of the source point.

In this paper, we address the registration problem to refine a pair of range surfaces as well as multi-view range surfaces. We propose an accurate and fast *point-to-plane*

registration technique. We combine advantages of *point-to-plane* and *point-to-projection* techniques for fast control point searching. We employ the accuracy from the *point-to-plane* technique and the speed from the *point-to-projection* technique. In order to find the intersecting control point, we project a source point to the destination surface, re-project the projection point to the normal vector of the source point. We show that iterative normal projections converge to the intersection point. By assuming the destination surface as a monotonic function in a new 2D coordinate system, we show a contraction mapping property of our registration technique [14]. Therefore, we call our technique as *Contractive Projection Point* (CPP) algorithm. Experimental results for several 3D models are presented for many single pair registrations as well as a multi-view registration.

2. Description of Registration Techniques

In this section, we briefly describe the basic principle of three registration refinement techniques. Suppose there is a control point set \mathbf{P} on a source surface \mathcal{S}_P , and another control point set \mathbf{Q} on a destination surface \mathcal{S}_Q . If we have K control points on each surface and $k = 0, \dots, K - 1$, then the registration problem is estimating a rigid body transformation $\mathbf{T} = [\mathbf{R}|\mathbf{t}]$ which minimizes an alignment error measure ϵ such that

$$\epsilon = \sum_{k=1}^K \|\mathbf{Q}_k - (\mathbf{R}\mathbf{P}_k + \mathbf{t})\|^2. \quad (1)$$

In general, source control point set \mathbf{P} is selected by sampling the source surface-randomly or uniformly, and filtered by some constraints to delete unreliable control points. The destination control point set \mathbf{Q} is the conjugate of the source point set, which is determined by a matching criterion.

Point-to-point technique (or ICP technique) is the most common technique. From a source control point \mathbf{p} on the source surface, the ICP algorithm searches the closest point \mathbf{q} on the destination surface. Figure 1(a) shows a basic diagram of the ICP algorithm. An error metric d_s is the distance between two control points. In order to search the closest point, a searching technique, e.g. kd-tree, is commonly used. However, when a multi-view registration concerned, we need to rebuild kd-trees of the multiple surfaces at every iteration of registration.

Point-to-plane registration is another common technique. It searches the intersection on the destination surface from the normal vector of the source point. As shown in the Figure 1(b), the destination control point \mathbf{q}' is the projection of \mathbf{p} onto the tangent plane at \mathbf{q} which is the intersection from the normal of \mathbf{p} . One of the previous investigations is done by Chen and Medioni [8]. To find control points, they use a root searching technique similar to Newton-Raphson

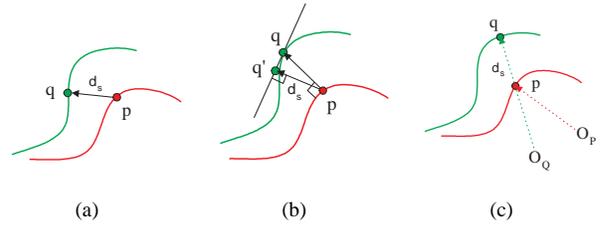


Figure 1. Three common registration techniques. (a) Point-to-point (b) Point-to-plane (c) Point-to-projection

technique in an orthographic coordinate system. Bergevin et al. [3] also use a similar technique with that of Chen and Medioni. They employ an image-based control point searching technique to reduce searching time.

Point-to-projection approach is known to be a fast registration technique. As shown in the Figure 1(c), this approach determines a point \mathbf{q} which is the conjugate of a source point \mathbf{p} , by forward-projecting \mathbf{p} from the point of view of the destination \mathbf{O}_Q . In order to determine the projection point, \mathbf{p} is first backward-projected to a 2D point \mathbf{p}_Q on the range image plane of the destination surface, and then \mathbf{p}_Q is forward-projected to the destination surface to get \mathbf{q} . This algorithm is very fast because it does not include any searching step to find the correspondence. However, one of its disadvantages is that the result of registration is not as accurate as those of the others [17].

3. Contractive Projection Point (CPP) Technique

3.1. Combining point-to-plane and point-to-projection techniques

In this section, we propose a novel *point-to-plane* registration technique by combining the fast registration property of the *point-to-projection* technique. Suppose there are two partial surfaces \mathcal{S}_P and \mathcal{S}_Q as shown in Figure 2. They are assumed to be acquired from two different views P and Q of an object and coarsely registered to a common (world) coordinate system through transformation matrices \mathbf{T}_P and \mathbf{T}_Q , respectively. Let us also suppose there is a source control point \mathbf{p}_0 on \mathcal{S}_P . Then the problem in the *point-to-plane* registration is finding the intersection point \mathbf{q}_s on \mathcal{S}_Q as shown in the figure. The point \mathbf{q}_s is an intersection on \mathcal{S}_Q by the normal vector $\hat{\mathbf{p}}$ of \mathbf{p}_0 .

One of the typical methods of searching the intersection is first finding a triangle (when the surface consists of tri-

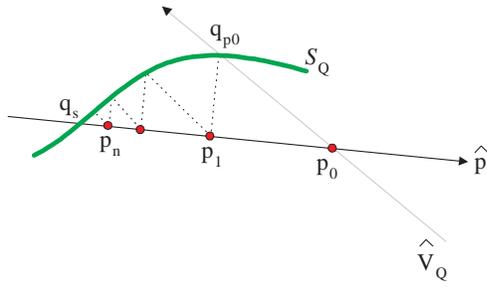


Figure 3. Contraction mapping property of a new 2D coordinate system.

3. If there is an intersection \mathbf{q}_s , a function $f : \mathbf{p}_i \rightarrow \mathbf{p}_{i+1}$ (or $\mathbf{q}_{p_i} \rightarrow \mathbf{q}_{p_{i+1}}$) is a contraction mapping when $\|\mathbf{p}_{i+1}, \mathbf{p}_{i+2}\| < k\|\mathbf{p}_i, \mathbf{p}_{i+1}\|$, where $0 \leq k < 1$.
4. Then, there is a 3D point \mathbf{q}_s on the surface such that $f(\mathbf{q}_s) = \mathbf{q}_s$.

3.3. Convergence condition

In the ideal situation, the CPP algorithm always converges to a convergence point. However, in a real situation, it may diverge or enter to a non-convergence cycle. Therefore we need to find out the convergence condition according to the contraction coefficient k . Three possible cases are shown in the Figure 4. In the Figure 4(a), a source control point is converging and the coefficient is $0 \leq k < 1$. In this case, we can use the convergence point as a matching control point of its conjugate.

However, as shown in the Figure 4(b), the control point could diverge if the coefficient is $k > 1$. This could happen when the tangent normal at a point on the surface S_Q has a high angle with respect to $\hat{\mathbf{V}}_Q$. However, we reduce the effect of diverging control points by discarding a destination point if the convergence measure ϵ_c is greater than an initial measure $\|\mathbf{q}_{p_0} - \mathbf{p}_0\|$. The last case is when the mapping enters into a non-convergence cycle and $k = 0$, as shown in the Figure 4(c). This case could happen when a segment of the destination curve is overlapping with the projection line from \mathbf{q}_{p_i} to \mathbf{p}_{i+1} . We also remove this kind of points by checking the error measure for 3 or 4 iterations. If a measured value repeats for 3 or 4 iterations, we consider the mapping enters a non-convergence cycle and quit the mapping.

3.4. CPP algorithm

The proposed CPP algorithm can be easily implemented by a recursive searching program. The following function is a pseudo code of part of the algorithm. The function

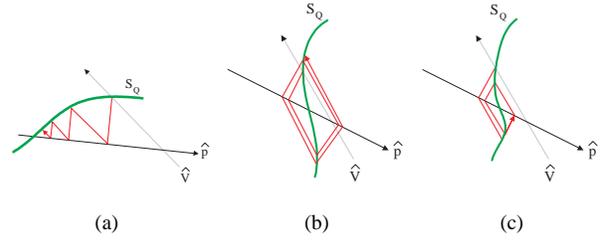


Figure 4. Three cases of projection. (a) Convergence ($0 \leq k < 1$) (b) Divergence ($k > 1$) (c) Infinity loop ($k = 1$)

searches a 3D point \mathbf{q}_{p_1} , the intersection of a control point \mathbf{p}_1 on the destination surface $Dest$. N_c is the maximum normal projections and D_c is threshold for the convergence error ϵ_c .

```

Vec3d RecursiveProj(Vec3d & p1, Vec3d & p,
    Vec3d & q_p1, Mesh * Dest, int cnt)
{
    if (cnt > N_c) return p1;
    p_q = M_q p1;
    flag = OnObject(p_q, p1, Dest, q_p1);
    if (!flag) return NULL;
    alpha = (q_p1 - p1) . p;
    p2 = p1 + alpha p;
    epsilon_c' = ||p2 - q_p1||;
    if (epsilon_c' < D_c) return p2;
    return RecursiveProj(p2, p, q_p1, Dest, cnt + 1);
}

```

After finding all matching control points, we use two control point sets $\{\mathbf{p}_0\}$ and $\{\mathbf{q}_s\}$ for estimating a rigid transformation \mathbf{T} between two surfaces. The transformation is computed by a SVD (Singular Value Decomposition) technique iteratively until two surfaces converge [1].

4. Experimental Results

4.1. Test objects

We test our algorithm for three objects. The test objects are shown in the Figure 5. The *Wave* object is a pair of synthetic 3D sinusoidal surfaces. Two surfaces have 10 degree of phase difference in the XY range image plane and 10mm of translation along the Z axis. Their height is 50mm and we add random noise to every image point with

maximum 12% of the height. The second object *Angel* is a pair of range images obtained from a laser range finder. This object is one of the models from the Ohio State University’s Range Image Database. The two images has 20 degree of rotation angle. For the third object *Potatohead* in Figure 5(c), we test both pair-wise and multi-view registration. We obtain 8 range images of the object. Range images of the object have some erroneous points on their surfaces. The number of points and triangles on the test objects are shown in the Table 1. They are the number of points and triangles on the surface of the first view, but the numbers of the others are also similar.

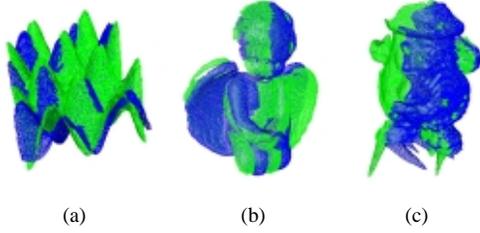


Figure 5. Point clouds models of test objects. (a) Wave (b) Angel (c) Potatohead

Table 1. Number of vertices and triangles on test objects

Objects	Wave	Angel	Potatohead
Num points (V_0)	22500	14089	13827
Num triangles (V_0)	44402	27213	27144
No. of views	2	2	8

4.2. Registration error with respect to projection numbers

In order to decide an appropriate number of normal projections – from \mathbf{q}_{pi} to \mathbf{p}_{i+1} , we apply the CPP technique to a noiseless *Wave* object to plot convergence error ϵ_c with respect to different numbers of projections. In Figure 6, the convergence error is plotted for 50 iterations of registration. At each iteration, RMS errors of all control points are plotted, for which we use about 300 pairs. As shown in the figure, the convergence error decreases if the number of projections increases. However, its decrease rate is saturated after 5 or 6 projections. Therefore, we use 5 normal projections for all following experiments.

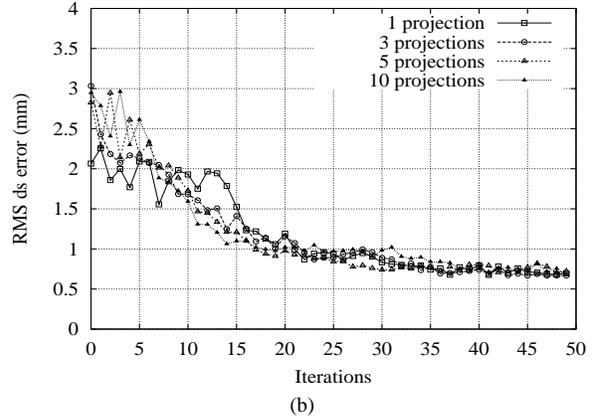
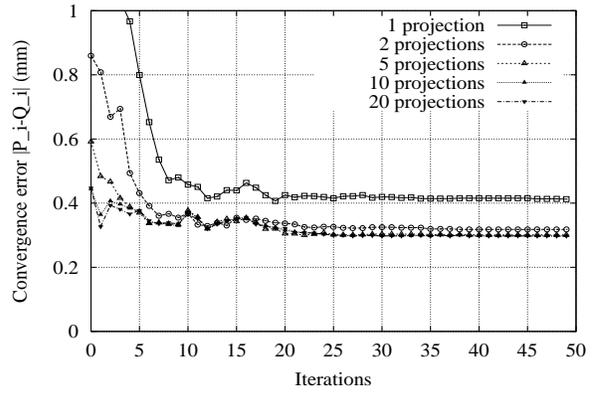


Figure 6. Comparison of convergence and translation error with respect to different number of projections. (*Wave* object) (a) Convergence (b) RMS translation

4.3. Pair-wise registration

We compare our algorithm with the *Point-to-Projection* and the ICP algorithm. We also use a kd-tree searching technique for ICP. We test all three techniques in the same registration condition on a Pentium III 1GHz personal computer. As far as the *Wave* object is concerned, we know the real conjugate points between two range surfaces. Therefore, we measure RMS distance error between two ground truth point sets. For the other objects, we measure d_s distance shown in the Figure 1. However, the distance error in each approach cannot be compared directly because they use different error metric. To fairly compare them, we normalize each error by using its initial error. Because the initial condition of all approaches is the same each other, we normalize the RMS errors to watch the decrease rate of them.

Registration results of CPP and ICP for *Wave* object are plotted in Figure 7. We plot the results for 50 iterations, and for different noise rate on the object's surface. Two algorithms converge well with low noise rate, however the ICP algorithm fails to converge when noise rate is more than 9%. The results of the other objects are shown in the Figure 8. Figure 8(a) shows the results for the object *Angel*. The *Point-to-projection* result shows a bad registration result. The proposed technique shows better convergence rate than other techniques. Figure 8(b) is the results for *Potato-head*. This result only shows the registration of the first pair of images. In this figure, we also plot the result of original point-to-plane technique, which use a brute force searching to find matching control points. The result of the proposed technique is almost the same as that of the original technique.

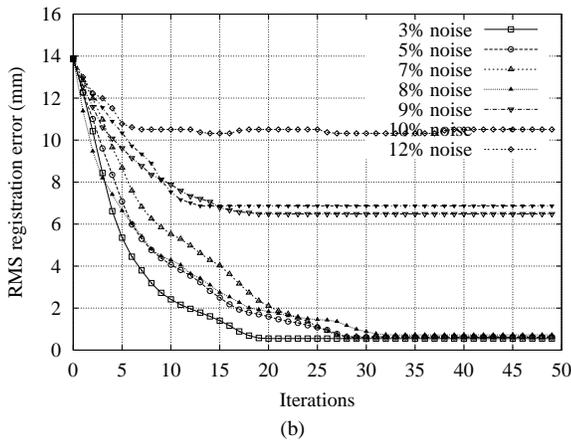
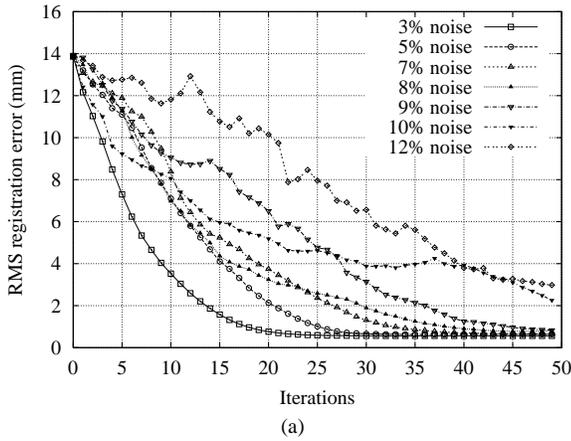


Figure 7. Registration error of *Wave* object (a) CPP (b) ICP

Table 2 shows the results of registration error and pro-

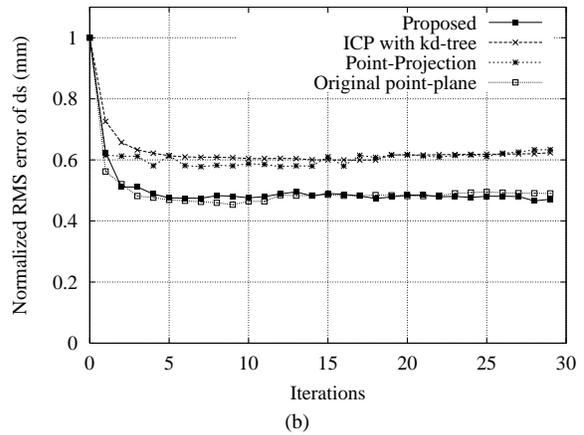
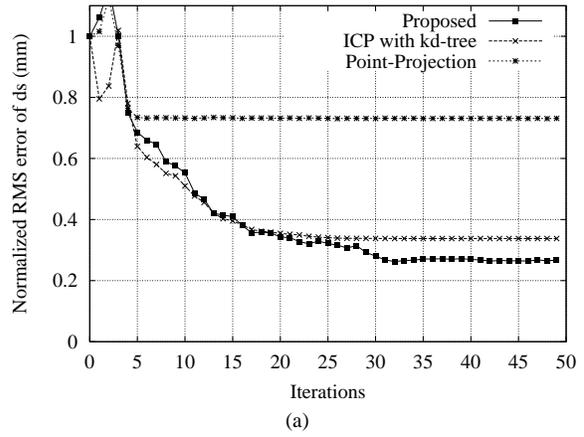


Figure 8. RMS error results of (a) *Angel* (b) *Potato-head*

cessing time after 50 iterations. As shown in the previous figure, our CPP technique has the smallest registration error. As far as the computation time is concerned, the point-to-projection technique is faster than the other approaches as we expected. The proposed technique and the point-to-point technique show similar results but they depend on types of objects. This is because we need some computations for searching intersection points as shown in the CPP pseudo code.

4.4. Number of convergence

As described in an earlier section, we have three types of convergence – Convergence, Divergence, and Infinity loop – in searching of intersections. After some experiments, we find that CPP searching converges well in most cases to intersections on destination surface. However, sometimes it

Table 2. Registration error and processing time after 50 iterations ($N_c = 5, D_c = 0.1mm$, and d_s is normalized.)

Method		PP	ICP	CPP
Wave with	ground truth(mm)	-	0.63	0.63
5% noise	Time/Itr(ms)	-	42.8	43.8
Wave with	ground truth(mm)	-	6.86	2.25
10% noise	Time/Itr(ms)	-	44.0	42.8
Angel	d_s (mm)	0.73	0.34	0.27
	Time/Itr(ms)	30.8	37.4	36.2
Potato-head	d_s (mm)	0.63	0.63	0.47
	Time/Itr(ms)	26.4	30.6	30.8

PP: Point-Projection, ICP: ICP with kd-tree, CPP: Contractive Projection Point

diverges or enters to an infinity loop. Even though we have some number of non-convergence points, we discard them and use only convergent control points for deriving transformation matrix. The numbers of converging and non-converging points are different for every iteration. However, Table 3 shows average numbers of them for 50 iterations on each object.

Table 3. Average number of convergence and non-convergence control points

Object	Convergence	Divergence	Infinity loop
Wave(5% noise)	92.5	6.2	1.3
Angel	220.5	52.7	2.6
Potatohead	202.1	32.9	3.2

4.5. Multi-view registration

Multi-view registration is a more difficult problem than a pair-wise registration, because registration error should be evenly distributed on all overlapping surfaces. We regard the first view of the *Potatohead* object as a reference view and register all the other views to the reference view. In order to find the matching control points from a source view, we search on its neighboring view’s surfaces. For example, if the i th view is concerned, we search the matching control points on its neighborhoods, $(i + 1)$ th view and $(i - 1)$ th view. After finding all matching points for every view point, except the first view, we compute rigid transformations and refine all surfaces simultaneously.

We test a multi-view registration for the 8 views of the *Potatohead* object. The Figure 9 plots their results. In this figure we do not normalize the error, because the results are

obvious to compare. Both the *point-to-projection* and the ICP techniques show bad registration results. Both of them fail to register the multi-view images. In contrast, the CPP algorithm register all range images well. The table 4 shows registration error and processing time for this experiment. The ICP with kd-tree technique take more time, because it needs to refine kd-trees of all surface (except the reference view) for every iteration. The table shows that our CPP technique has advantages over ICP and *Point-to-projection* techniques for registration of multiple range images.

Table 4. Multi-view registration error and processing time after 50 iterations. ($N_c = 5, D_c = 0.1mm$)

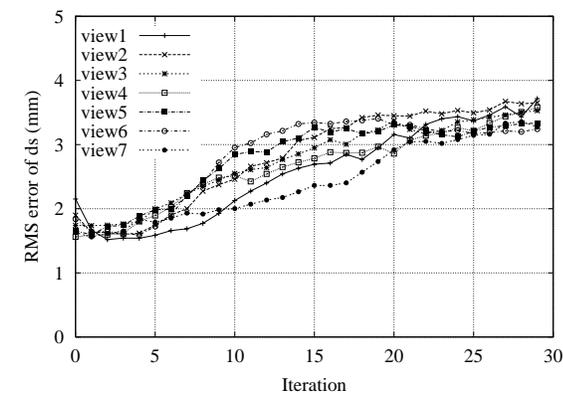
Method	PP	ICP	CPP
RMS error d_s (mm)	3.45	1.88	0.74
Time (sec)	9.61	46.57	12.63
Time/Itr (sec)	0.192	0.931	0.253

5. Conclusions

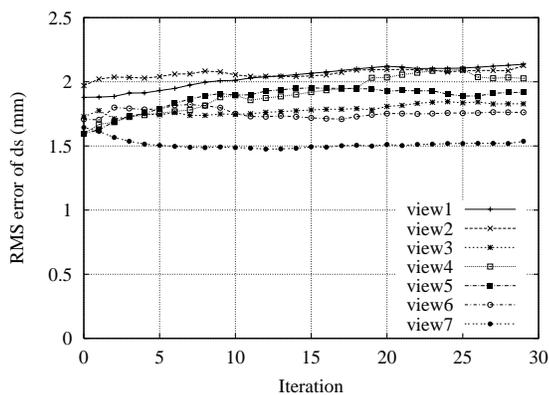
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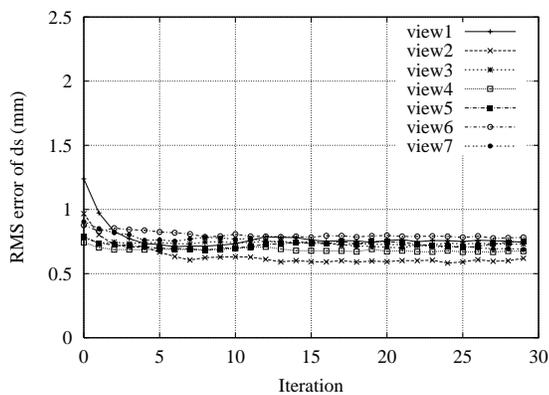
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(a)



(b)



(c)

Figure 9. Results of multi-view registration for *Potatohead* after 30 iterations. (a) Point-to-projection (b) ICP with kd-tree (c) Proposed (CPP)

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