packets. The expected data-packet delay of the contention-based movable-boundary WIMA protocol in log-normal-fading channel increases with increasing $\sigma_{w}$.

## V. Conclusion

In this paper, the performance of a mobile radio network using contention-based movable-boundary WIMA protocol in Rayleighand log-normal-fading environments is simulated. The specifications of the DCS-1800 cellular mobile system are used to evaluate the voice-call dropping probabilities and the expected data-packet delay of the mobile radio network for different protocols in fading environments. The simulation results demonstrate that for high data arrival rate, the contention-based movable-boundary WIMA protocol can obviously improve the expected data-packet delay of the mobile radio network compared to the GPRS and fixed-boundary WIMA protocols in the Rayleigh-fading environment. The expected data-packet delay of the contention-based movable-boundary WIMA protocol will be increased rapidly when data arrival rate exceeds the threshold value. The voice-call dropping probability and the expected data-packet delay of the contention-based movable-boundary WIMA protocol in the log-normal-fading channel increase with increasing variance of the channel fading. The analytical results validate the simulation results.

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# Multibeam Cellular Mobile Communications With Dynamic Channel Assignment 

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#### Abstract

In this paper, we consider sectorized multibeam cellular mobile communications with dynamic channel assignment to beams. Network performance for space-division multiple access (SDMA) based on channel reuse between beams is investigated. We use a model to calculate theoretical traffic performance characteristics of the proposed system including call-blocking probability and carried traffic.


Index Terms- Channel reuse, resource management, space-division multiple access (SDMA), traffic performance.

## I. Introduction

The demand for increased system capacity surges as the number of mobile users grows at rapid pace. Increased system capacity can be achieved by sectorization and cell splitting while limiting interference to maintain signal quality. One technique for increasing system capacity is space-division multiple access (SDMA), which uses directional antennas for reduction of cochannel interference and improvement of channel reuse [1], [2]. In SDMA, users in different angular positions can be served on the same channel if the angular separation between them is large enough [3]-[5].
One approach in SDMA is the switched multibeam system, where multiple beams are used to cover the entire coverage of the base station and the beam with the strongest signal power for the desired user is selected to serve the user. Recent work on switched multibeam systems is reported in [6], where gain improvement achieved with a multibeam antenna compared to the traditional sector configuration is investigated and tradeoffs between hysteresis level, switching time, and gain for a multibeam antenna system are considered. In [7], the effects of incorrect beam selection on average signal-to-noise ratio (SNR) and signal-to-interference ratio (SIR) with a switched multibeam antenna system are examined. The network performance of multibeam cellular systems including call blocking and forced termination probability is investigated in [8]. There, the advantage of combined multibeam scheme and dynamic channel assignment across multiple sectors are studied. The frequency reuse efficiency of multibeam systems is explored in [3] and [4].
It is well known that dynamic channel assignment (DCA) improves traffic performance in cellular communications [14], [15]. In DCA, all channels are stored in a common pool and are dynamically assigned to wireless users, subject to constraints on the allowable cochannel interference. In general, DCA falls into two categories: centralized and distributed. In centralized DCA, a radio network center maintains a table in its database to track the channel occupancy of each wireless gateway. Depending on the layout of wireless cellular systems, the wireless gateway could be a cell that is covered by omnidirectional antenna, a sector that is covered by sectored antenna, or a narrow direc-

[^0]tional beam. The assignment of channels at a gateway is dynamic, but follows the rule of minimum channel reuse distance. A channel can be assigned to a call at a gateway only when the same channel is not used in other gateways within the minimum reuse distance. If the channel assignment is made based on a channel occupancy condition of all the other gateways in the network, the DCA is referred to as fully centralized DCA. Fully centralized DCA has significant complexity and requires large signaling bandwidth. To simplify the DCA, locally centralized DCA is introduced, in which the decision of channel assignment is based only on the channel occupancy of neighboring gateways. An alternative to centralized DCA is distributed DCA, where the channel assignment to a user at a gateway is mostly based on the measurement reports of that gateway only. Available channels are the channels whose SIR is above a required threshold. The distributed DCA is less complex and requires smaller volume of signaling flow at network level, but heavily relies on the accuracy of interference measurement.
In this paper, a switched multibeam system is considered, where system capacity is increased by reusing channels in different beams whenever cochannel interference remains below a specified level. We consider a locally centralized DCA within a sector, and we compare sectorized cellular systems with and without multibeam schemes. We show that for a fixed offered traffic, the blocking probability of calls can be reduced significantly, or alternatively, that more new call traffic can be supported while the blocking probability is maintained. As the number of beams increases and more channel reuse is allowed, the traffic performance improves. However, the switching or handoff traffic from one beam to another increases as well. Due to the narrower beams, mobile stations easily move beyond their current serving beams, and as a result, the link quality may deteriorate, primarily due to increased interference arising from intensified channel reuse. As a result, there is a tradeoff between number of beams, system capacity, network signaling performance, and link quality.

We compare the performances of multibeam schemes with and without channel rearrangement. We devise models to compute fundamental traffic performance measures, including call-blocking probability, carried traffic and channel rearrangement rate for multibeam cellular systems. The models are based on multidimensional birth-death processes [8]-[11]. We determine the global balance equations, solve for the state probabilities using a framework developed in earlier work [9], [10], and find the performance characteristics from the state probabilities. In Section II, we present the analytical model for multibeam cellular communication systems with dynamic channel assignment, where as an example, a multibeam system with three beams per sector and three sectors per cell is considered. In Section III, we describe the state representation, and in Section IV, we explain the driving processes and state balance equations. Generalization of the model to systems with a larger number of beams and sectors is given in Section V. At last, in Sections VI and VII, we discuss numerical results and provide conclusions.

## II. Model Description

We consider a switched multibeam system, in which cells are each divided into several sectors and each sector is covered by several directional beams. Certain channels are allocated to each sector, and the channels assigned to a sector can be reused in different beams of the sector provided that a required angular separation between beams is met. A channel could be a frequency band for frequency division multiple access (FDMA), a time slot for time division multiple access (TDMA), a spreading or scrambling code for code division multiple access (CDMA), or any combination of them. Note that the addressed systems do not correspond to any existing second- and third-generation systems. The methodology in this paper, however, can easily be modi-


Fig. 1. A beam layout of $120^{\circ}$-sectorized nine-beam scheme.
fied to reflect the specificity of any particular system of interest. In our model, we do not consider angular spreading due to scattering effects.

A locally centralized DCA within a sector is considered. A common pool of channels is allocated to each sector. No fixed set of channels is allocated to each beam. Assignment of channel resources to a beam is made based on the channel occupancy conditions of its neighboring beams of the same sector without regard to the channel usage pattern of other beams in other sectors or cells. To allow for more efficient use of channel resources and avoidance of unnecessary call blocking, DCA is usually combined with channel rearrangement. In channel rearrangement the channel used to serve a particular call is not fixed. In fact, depending on the channel occupation, a call may switch between several different channels during its lifetime, and as a result, system implementation with channel rearrangement is much more complex.

## A. An Example of a System Model

An example model that we consider is a $120^{\circ}$-sectored (three-sector) multibeam cellular system with three beams deployed in each sector, which provides a total of nine beams per cell. Each sector has a limit of $C$ channels and consists of one center beam and two side beams, where the beams are numbered in a counterclockwise direction as shown in Fig. 1. For example, beam 2 is a center beam and beams 1 and 3 are two side beams in sector 1 . The cochannel interference in a given beam at a site arises from the use of the same channel in other beams at the same site as well as from the use of the same channel at other sites. The channels can be reused in the two side beams if the minimum angular separation between them is large enough so that the overall cochannel interference is below a required level.

## B. Channel Assignment

DCA to beams within sectors is used in order to utilize channel resources of the system efficiently. For DCA in the present context, there is no fixed set of channels allocated to each beam. The channels of a sector are dynamically assigned to wireless users in that sector without regard to the beam through which they communicate as long as the assignment of channel resources follows the required rule. Thus, the DCA is locally centralized within a sector and channels are assigned based on the channel occupancy condition of beams in that sector only. Channel assignment is made in a way that no adjacent beams of a sector can use the same channels at the same time. It is assumed that the interference caused to or coming from other beams is maintained below the
required level for as long as the maximum number of supported calls by the DCA is not exceeded. To ensure the maximum channel reuse, channel assignment is made by the following criteria:

- if at the time of a new call arrival in a side beam, there is an available channel that is already in use in the other side beam, this channel is assigned to serve the new call;
- if there is no such channel in the side beam, then an available channel is randomly selected to serve the new call.
Thus, channels that are allocated to the sector can be reused in the side beams with maximum capacity.
When at the time of a new call arrival in the center beam all the channels in the sector are occupied and a channel rearrangement is impossible, the call is blocked. Channel rearrangements can take place only if there are at least two channels available, one of which is in use in beam 1 but not in use in beam 3, and vice versa. Then, a rearrangement is made so that one of the two channels is reused to serve the two existing calls in beams 1 and 3 , while the other channel is used to serve the new call in the center beam.

Note that in this paper, the maximum number of calls that a DCA can support is fixed. This number can be determined by the worst case SIR link quality. In general, an approach using dynamic SIR can be developed that may have improved performance. Its implementation complexity, however, may increase as well.

## III. State Description

In this section, we consider an infinite-population model, with single platforms, single calls, and three beams per sector. We define the state of each sector by four state variables $\nu_{1}, \nu_{2}, \nu_{3}$, and $\alpha$, where $\nu_{i}$, $i=1,2,3$, is the number of calls served by the $i$ th beam, and $\alpha$ is the number of identical channels in use in beams 1 and 3 . We order the states using the index $s=0,1,2, \ldots, S_{\text {max }}$, where $S_{\text {max }}$ is the maximum state index. Ordering the states by index allows us to express $\nu_{i}$, $i=1,2,3$, and $\alpha$ as functions of the state. More specifically, we can write $\nu_{i}=\nu(s, i), i=1,2,3$, where $\nu(s, i)$ is the number of calls served by the $i$ th beam when the sector is in state $s$, and $\alpha(s)$ is the number of channels in use in beams 1 and 3 that are the same when the sector is in state $s$.
Channel assignment follows the rule that no adjacent beams can use the same channels at the same time. This assignment rule of channels ensures that the same channels in use in the same sector are served by beams with enough separation in angular space and that interference is maintained below the required level. The maximum number of calls that can be supported in a beam without causing interference to other beams and sectors above the required level is based on a worst case scenario. In the worst case, all beams of all sectors are occupied by the maximum number of calls. This number can be determined for a specified interference level. We assume that a new call can be accommodated without violating the interference requirement of its own and causing excessive interference to other calls as long as the number of active calls in a beam has not reached the maximum limit. Let $C$ denote the number of channels in each sector. We can specify the constraints on permissible states as

$$
\begin{align*}
\nu(s, i)+\nu(s, i+1) & \leq C, \quad i=1,2  \tag{1}\\
\sum_{i=1}^{3} \nu(s, i)-\alpha(s) & \leq C  \tag{2}\\
\alpha(s) & \leq \min [\nu(s, 1), \nu(s, 3)] \tag{3}
\end{align*}
$$

The constraints given by (1) mean that the number of channels in use in any two adjacent beams of a sector when the sector is in state $s$ cannot be larger than $C$. The constraint given by (2) indicates that the total number of different channels in use in a sector when the sector is in state
$s$ cannot be more than $C$. Finally, (3) states that the number of identical channels in use in beams 1 and 3 cannot exceed the smaller number of channels in one of the side beams. The number of calls served in each sector can be larger than the number of channels in the sector because of the channel reuse. A sector in a multibeam scheme with three beams per sector and channel reuse within nonadjacent beams of the sector can support at most $2 C$ calls.

## IV. Driving Processes and State Transition Flow

To determine the performance measures of interest, we need to find the state probabilities $p(s)$ in statistical equilibrium. The computation of the state transitions requires identification and calculation of the corresponding transitions. There are two relevant driving processes: one is the generation of new calls in a sector $\{n\}$ and the other, the completion of calls in a sector $\{c\}$. We make the assumptions that the new call arrival processes in any state follow the Poisson point process, and that the unencumbered call duration of a call has a negative exponential distribution. The transition rates into a state $s$ from a predecessor state $x$ due to a new call arrival in beam $i$ are denoted by $\gamma_{n i}(s, x)$, $i=1,2,3$, respectively. Similarly, the transition rates into a state $s$ from predecessor state $x$ due to a call completion in beam $i$ are denoted by $\gamma_{c i}(s, x), i=1,2,3$, respectively. All the driving processes and corresponding state transition flows are explained in the Appendix.
The total transition flow into $s$ from any permissible predecessor state $x$ can be found using

$$
\begin{align*}
& q(s, x)=\gamma_{n 1}(s, x)+\gamma_{n 2}(s, x)+\gamma_{n 3}(s, x) \\
&+\gamma_{c 1}(s, x)+\gamma_{c 2}(s, x)+\gamma_{c 3}(s, x) \tag{4}
\end{align*}
$$

where $s \neq x$, and the flow into a state has been taken as a positive quantity. The total flow out of a state $s$ is denoted as $q(s, s)$, and is given by

$$
\begin{equation*}
q(s, s)=-\sum_{\substack{k=0 \\ k \neq s}}^{S_{\max }} q(k, s) \tag{5}
\end{equation*}
$$

To find the state probabilities for a sector in statistical equilibrium, we write the flow balance equations for the states and solve them. They represent a set of $S_{\max +1}$ equations in the unknown state probabilities $p(s)$, and have the form

$$
\begin{align*}
\sum_{j=0}^{S_{\max }} q(i, j) p(j) & =0, \quad i=0,1, \ldots, S_{\max }-1 \\
\sum_{j=0}^{S_{\max }} p(j) & =1 \tag{6}
\end{align*}
$$

where for $i \neq j, q(i, j)$ represents the net transition flow into state $i$ from state $j$, and $q(i, i)$ is the total transition flow out of state $i$. These equations simply state that in statistical equilibrium the net probability flow into any state is zero, and the sum of the probabilities is unity.

## V. Generalization

The model can be generalized to a larger number of beams and sectors. It is assumed that DCA is made by the rule that no adjacent beams of a sector can use the same channels at the same time. In the previous sections, the state variable $\alpha$ is introduced to track the state of a sector whenever a channel rearrangement is performed. In this section, for simplicity, no state variables are used for this purpose, and it is assumed that channel rearrangement is always executed whenever necessary. This assumption does not affect the major performance metric: the blocking probability.

## A. State Description

For the problem under consideration in this section (single-platform type, single-call type, $K$ beams per sector, infinite population model), $K$ state variables, $\nu_{i}, i=1,2, \ldots, K$, are needed to define the state of a sector. Again, the state variable, $\nu_{i}$, is the number of calls served by the $i$ th beam, and the states are ordered using an index $s=0,1,2, \ldots, S_{\max }$. Also, $\nu_{i}, i=1,2, \ldots, K$ can be denoted as explicitly dependent on the state, or $\nu_{i}=\nu(s, i), i=1,2, \ldots, K$.

If $C$ denotes the number of channels in each sector, we can specify the constraints on permissible states as

$$
\begin{equation*}
\nu(s, i)+\nu(s, i+1) \leq C, \quad i=1,2, \ldots, K-1 \tag{7}
\end{equation*}
$$

which means that the number of channels in use in any two adjacent beams of a sector when the sector is in state $s$, cannot be greater than $C$.

## B. Driving Processes and State Transition Flow

We now describe the state transition flow due to the driving processes, the new call arrivals and call completions.

1) New Call Arrivals: A transition into the state $s$ due to a new call arrival in beam $i$ when the sector is in state $x_{n}$ will cause the state variable $\nu\left(x_{n}, i\right)$ to be incremented by one. A new call can be served in beam $i$ only if the number of channels in use in any two adjacent beams does not exceed $C$. Thus, a permissible state $x_{n}$ is a predecessor state of $s$ for new call arrivals in beam $i$, if

$$
\nu\left(x_{n}, i\right)+\max \left[\nu\left(x_{n}, i-1\right), \nu\left(x_{n}, i+1\right)\right]<C
$$

and the state variables are related by

$$
\begin{align*}
\nu\left(x_{n}, i\right) & =\nu(s, i)-1 \\
\nu\left(x_{n}, j\right) & =\nu(s, j), \quad j \neq i \tag{8}
\end{align*}
$$

If $\Lambda_{n i}, i=1,2, \ldots, K$ is the average arrival rate of new calls in beam $i$, the flow into state $s$ from $x_{n}$ due to new call arrivals in beam $i$ is

$$
\gamma_{n i}\left(s, x_{n}\right)=\Lambda_{n i}
$$

if

$$
\begin{equation*}
\nu\left(x_{n}, i\right)+\max \left[\nu\left(x_{n}, i-1\right), \nu\left(x_{n}, i+1\right)\right]<C . \tag{9}
\end{equation*}
$$

2) Call Completions: When there is a call completion, the transition into state $s$ in beam $i$, provided the sector is in state $x_{c}$, will cause the state variable $\nu\left(x_{c}, i\right)$ to decrease by one. As a result, a permissible state $x_{c}$ is a predecessor state of $s$ for call completion in beam $i$, if the state variables are related by

$$
\begin{align*}
\nu\left(x_{c}, i\right) & =\nu(s, i)+1 \\
\nu\left(x_{c}, j\right) & =\nu(s, j), \quad j \neq i \tag{10}
\end{align*}
$$

If we denote the average completion rate of each call by $\mu_{c}$, for the flow into state $s$ from $x_{c}$ due to call completion in beam $i$, we can write

$$
\begin{equation*}
\gamma_{c i}\left(s, x_{c}\right)=\nu\left(x_{c}, i\right) \mu_{c} . \tag{11}
\end{equation*}
$$

## C. Performance Measures

Once the statistic equilibrium state probabilities are found, the required performance measures can be calculated.

1) Blocking Probability: The blocking probability for a call is the average fraction of new calls that are denied access to a channel.

Blocking of new calls occurs if there are no channels to serve the call. We define the following sets of states:

$$
\begin{align*}
B_{1} & =\{s: \nu(s, 1)+\nu(s, 2)=C\}  \tag{12}\\
B_{i} & =\{s: \nu(s, i)+\max [\nu(s, i-1), \nu(s, i+1)]=C\}  \tag{13}\\
B_{k} & =\{s: \nu(s, K-1)+\nu(s, K)=C\}
\end{align*}
$$

which are used to write succinctly the blocking probability in a beam. For beam $i$, this probability, $P_{b i}$, is calculated according to

$$
\begin{equation*}
P_{b i}=\sum_{s \in B_{i}} p(s), \quad i=1,2, \ldots, K \tag{15}
\end{equation*}
$$

Then, the average blocking probability in a sector can be obtained from

$$
\begin{equation*}
P_{B}=\sum_{i=1}^{K} f_{i} P_{b i} \tag{16}
\end{equation*}
$$

where $f_{i}$ is the average fraction of new calls that arrive in beam $i$ of the sector.
2) Carried Traffic: The carried traffic for a sector is defined as the average number of channels occupied by the calls. The carried traffic in beam $i, A_{c i}$ is evaluated by

$$
\begin{equation*}
A_{c i}=\sum_{s=0}^{S_{\max }} \nu(s, i) p(s), \quad i=1,2, \ldots, K \tag{17}
\end{equation*}
$$

and the overall carried traffic of a sector $A_{C}$ is given by

$$
\begin{equation*}
A_{C}=\sum_{i=1}^{K} A_{c i} \tag{18}
\end{equation*}
$$

3) Channel Rearrangement Rate: Channel rearrangement rate is the average rate of channels that must be rearranged. Here, the channel rearrangement rate is defined for the case of $K=3$ only, and for $K>3$, it is assumed that channels are always rearranged whenever necessary. To describe the channel rearrangement, we define the set of states $W$ by

$$
\begin{align*}
W=\{s: \nu(s, 2)+\max [\nu(s, 1), & \nu(s, 3)]<C \\
& \left.\sum_{i=1}^{3} \nu(s, i)-\alpha(s)=C\right\} \tag{19}
\end{align*}
$$

The channel rearrangement rate $R$ is defined as the product of the rate of new call arrivals in the center beam and the probability that the system is in a state that would require channel rearrangement, i.e.,

$$
\begin{equation*}
R=\Lambda_{n 2} \sum_{s \in W} p(s) \tag{20}
\end{equation*}
$$

## VI. Numerical Results

In this section, we provide some results that describe the performance of the systems with dynamic channel assignment. The mean unencumbered call duration time of a call was 100 s , which corresponds to average call completion rate $\mu_{c}$ of 0.01 calls $/ \mathrm{s}$. Each sector had fifteen channels $C=15$, and the new call origination in a sector was uniformly distributed. The beams are symmetric with respect to their centers and are identical, and they have equal widths. Thus, the traffic of new call in each beam is the same.


Fig. 2. Dependence of blocking probability on demand. There were 15 channels per sector. The beams had equal beamwidths, and the mean call duration was 100 s .


Fig. 3. Dependence of carried traffic on demand. There were 15 channels per sector. The beams had equal beamwidths, and the mean call duration was 100 s .

The new call origination rates in a sector were varied from 0.08 to 0.12 calls/s. In Figs. 2 and 3, we show the dependence of blocking probability and carried traffic as a function of the new call origination rate, respectively. Three schemes were compared-the conventional three-sector, the nine-beam, and the fifteen-beam schemes. In terms of blocking probability, it is obvious that the scheme using multiple beams significantly outperforms the scheme without multiple beams. The call-blocking performance of the multibeam scheme is improved significantly as the number of beams per sector is increased. This is due to the channel reuse between beams. Also, for a fixed offered traffic, the blocking probability is reduced significantly. Alternatively, more new call traffic can be supported while the blocking probability is maintained. In summary, Fig. 2 shows that the blocking probabilities are greatly reduced by the use of multiple beams in a sector. Fig. 3 displays that the carried traffic for the three schemes is almost the same (it increases slightly with the call origination rate). This is an advantage of the multibeam scheme, which allows an overall traffic improvement of the major performance metrics. The cost is increased complexity of implementation in comparison with the conventional scheme that does not use multiple beams.


Fig. 4. Comparison between multibeam schemes with and without channel rearrangement. There were 15 channels per sector. The beams had equal beamwidths, and the mean call duration was 100 s .


Fig. 5. Dependence of channel rearrangement rate on demand. There were 15 channels per sector. The beams had equal beamwidths, and the mean call duration was 100 s .

In Fig. 4, the blocking probabilities of a nine-beam scheme with and without channel rearrangement are compared. In a nine-beam scheme, the blocking probability in the center beam is worse than that in the side beams. To reduce the unnecessary blocking of new calls in the center beam and to balance the blocking probabilities between the center and side beams, channel rearrangement is used. Fig. 5 shows the channel rearrangement rate, where it is seen that the channel rearrangement rate increases as the new call origination rate increases.

## VII. Conclusion

In this paper, we considered sectorized multibeam cellular systems with multiple beams in each sector. Multibeam schemes provide significantly better blocking performance in comparison with schemes that do not use multiple beams in sectors. The blocking performance of multibeam schemes is further improved as the number of beams per sector is increased. Additional improvement in performance is achievable when multibeam schemes are combined with DCA.
In our work, we did not analyze the tradeoff between traffic performance and the SIR ratio. It is obvious that the SIR link quality of multibeam scheme depends on the actual number of beams deployed in
sectors as well as the channel assignment rule that drives DCA and the deployed system. In general, a multibeam scheme with fewer beams per sector has better SIR link quality due to less interference.

System capacity can be improved significantly by using multiple beams in sectors. More traffic can be accommodated while blocking probability of calls is maintained below the required level. Alternatively the blocking probability of new calls can be reduced for fixed offered traffic. The improvement in system capacity, however, is at the expense of poor link quality. Since the system capacity and SIR link quality are exchangeable by changing the number of beams in sectors, a tradeoff between system capacity and SIR link quality can be made. A system with moderate number of beams can achieve significant improvement in traffic performance while the link quality is maintained at an acceptable level.

## APPENDIX <br> Driving Processes and State Transition Flow

## 1) New Call Arrivals:

a) New call arrivals in beam $i$ : A transition into state $s$ due to a new call arrival in beam $i$ when the sector is in state $x_{n}$ will cause the state variable $\nu\left(x_{n}, i\right)$ to be incremented by one. A new call can be served in beam $i$ only if the number of channels in use in both beam $i$ and the center beam (beam 2) does not exceed $C$. If a new call arrives in beam $i$ when a channel is available in beam $i$ that is already in use in the other side beam (beam $k$ ), the channel is assigned to serve the new call. This causes the state variables $\alpha\left(x_{n}\right)$ and $\nu\left(x_{n}, i\right)$ to be incremented by one. Thus, a permissible state $x_{n}$ is a predecessor state of $s$ for new call arrivals in beam $i$, if $\nu\left(x_{n}, i\right)+\nu\left(x_{n}, 2\right)<C$, and $\nu\left(x_{n}, k\right)>$ $\alpha\left(x_{n}\right)$, and the state variables are related by

$$
\begin{align*}
\nu\left(x_{n}, i\right) & =\nu(s, i)-1 \\
\nu\left(x_{n}, j\right) & =\nu(s, j), \quad j \neq i \\
\alpha\left(x_{n}\right) & =\alpha(s)-1 . \tag{21}
\end{align*}
$$

If there is no channel available in beam $i$ and already in use in beam $k$, another available channel is selected randomly to serve the new call. This causes the state variable $\alpha\left(x_{n}\right)$ to remain unchanged and the state variable to be incremented by one. Thus, a permissible state $x_{n}$ is a predecessor state of $s$ for new call arrivals in beam $i$, if $\nu\left(x_{n}, i\right)+$ $\nu\left(x_{n}, 2\right)<C$ and $\nu\left(x_{n}, k\right)=\alpha\left(x_{n}\right)$, and the state variables are related by

$$
\begin{align*}
\nu\left(x_{n}, i\right) & =\nu(s, i)-1 \\
\nu\left(x_{n}, j\right) & =\nu(s, j), \quad j \neq i \\
\alpha\left(x_{n}\right) & =\alpha(s) . \tag{22}
\end{align*}
$$

Let $\Lambda_{n i}, i=1,2,3$ denote the average arrival rate of new calls in beam $i$. The flow into state $s$ from $x_{n}$ due to new call arrivals in beam $i$ is given by

$$
\gamma_{n i}\left(s, x_{n}\right)=\Lambda_{n i}
$$

if

$$
\begin{equation*}
\nu\left(x_{n}, i\right)+\nu\left(x_{n}, 2\right)<C \tag{23}
\end{equation*}
$$

b) New call arrivals in the center beam: A new call arrival in the center beam when the sector is in state $x_{n}$ will increase the state variable $\nu\left(x_{n}, 2\right)$ by one. The new call will be served in the center beam only if the total number of channels in the center and any of the side beams does not exceed $C$. When a new call arrives in the center beam and there is an available channel to serve the call, the state variable $\alpha\left(x_{n}\right)$ remains the same and the state variable $\nu\left(x_{n}, 2\right)$ is incremented
by one. So, a permissible state $x_{n}$ is a predecessor state of $s$ for new call arrivals in the center beam, if $\sum_{i=1}^{3} \nu\left(x_{n}, i\right)-\alpha\left(x_{n}\right)<C$. For the state variables, we can write

$$
\begin{align*}
\nu\left(x_{n}, 2\right) & =\nu(s, 2)-1 \\
\nu\left(x_{n}, j\right) & =\nu(s, j), \quad j \neq 2 \\
\alpha\left(x_{n}\right) & =\alpha(s) \tag{24}
\end{align*}
$$

When a new call arrives in the center beam and there is no available channel to serve the call, channel rearrangement is carried out. Then, the state variables $\alpha\left(x_{n}\right)$ and $\nu\left(x_{n}, 2\right)$ are incremented by one. An allowable state $x_{n}$ is a predecessor state of $s$ for new call arrivals in the center beam, if

$$
\begin{equation*}
\left.\nu\left(x_{n}, 2\right)+\max \left[\nu\left(x_{n}, 1\right), \nu\left(x_{n}, 3\right)\right)\right]<C \tag{25}
\end{equation*}
$$

and

$$
\begin{equation*}
\sum_{i=1}^{3} \nu\left(x_{n}, i\right)-\alpha\left(x_{n}\right)=C \tag{26}
\end{equation*}
$$

In this situation, the relationships between the state variables are

$$
\begin{align*}
\nu\left(x_{n}, 2\right) & =\nu(s, 2)-1 \\
\nu\left(x_{n}, j\right) & =\nu(s, j), \quad j \neq 2 \\
\alpha\left(x_{n}\right) & =\alpha(s)-1 \tag{27}
\end{align*}
$$

The flow into state $s$ from $x_{n}$ due to new call arrivals in the center beam is

$$
\gamma_{n 2}\left(s, x_{n}\right)=\Lambda_{n 2}
$$

if

$$
\begin{equation*}
\nu\left(x_{n}, 2\right)+\max \left[\nu\left(x_{n}, 1\right), \nu\left(x_{n}, 3\right)\right]<C \tag{28}
\end{equation*}
$$

## 2) Call Completion:

a) Call completions in beam $i, i=1,3$ : Suppose a call completion occurs in beam $i$ when the sector is in state $x_{c}$. Then, the state variable $\nu\left(x_{c}, i\right)$ decreases by one. If the channel serving the call is also in use in the other side beam, this call completion causes the state variable $\alpha\left(x_{c}\right)$ to decrease by one. A permissible state $x_{c}$ is a predecessor state of $s$ for call completion in beam $i$, if the state variables satisfy

$$
\begin{align*}
\nu\left(x_{c}, i\right) & =\nu(s, i)+1 \\
\nu\left(x_{c}, j\right) & =\nu(s, j), \quad j \neq i \\
\alpha\left(x_{c}\right) & =\alpha(s)+1 \tag{29}
\end{align*}
$$

Let $\mu_{c}$ denote the average completion rate of each call. Then, the flow into state $s$ from $x_{c}$ due to a call completion in beam $i$ is

$$
\begin{equation*}
\gamma_{c i}\left(s, x_{c}\right)=\alpha\left(x_{c}\right) \mu_{c} \tag{30}
\end{equation*}
$$

If the channel serving the call is not in use in the other side beam, this call completion entails the state variable $\alpha\left(x_{c}\right)$ to remain unchanged and the state variable $\nu\left(x_{c}, i\right)$ to decrease by one. Hence, an acceptable state $x_{c}$ is a predecessor state of $s$ for call completion in beam $i$, if the state variables fulfill the relationships

$$
\begin{align*}
\nu\left(x_{c}, i\right) & =\nu(s, i)+1 \\
\nu\left(x_{c}, j\right) & =\nu(s, j), \quad j \neq i \\
\alpha\left(x_{c}\right) & =\alpha(s) \tag{31}
\end{align*}
$$

The corresponding transition flow is given by

$$
\begin{equation*}
\gamma_{c i}\left(s, x_{c}\right)=\left(\nu\left(x_{c}, i\right)-\alpha\left(x_{c}\right)\right) \mu_{c} \tag{32}
\end{equation*}
$$

b) Call completions in the center beam: In the case of a call completion in the center beam when the sector is in state $x_{c}$, the state variable $\nu\left(x_{c}, 2\right)$ decreases by one and the state variable $\alpha\left(x_{c}\right)$ remains unchanged. A predecessor state of $s$ for a call completion in the center beam, $x_{c}$, is permissible if the state variables satisfy

$$
\begin{align*}
\nu\left(x_{c}, 2\right) & =\nu(s, 2)+1 \\
\nu\left(x_{c}, j\right) & =\nu(s, j), \quad j \neq 2 \\
\alpha\left(x_{c}\right) & =\alpha(s) . \tag{33}
\end{align*}
$$

The flow into state $s$ from $x_{c}$ as a result of call completion in the center beam is given by

$$
\begin{equation*}
\gamma_{c 2}\left(s, x_{c}\right)=\nu\left(x_{c}, 2\right) \mu_{c} \tag{34}
\end{equation*}
$$

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# Performance Analysis of Fiber-Fed Microcellular Networks Using $\pi / 4$-DQPSK in a Frequency-Selective CCI-Limited Nakagami Fading Environment 

G. S. Prabhu and P. M. Shankar


#### Abstract

The use of wide-band and low-loss fiber-optic feeders to transfer the complex signal processing and control functions from the base station (BS) to a central control station (CCS) in a microcellular system has recently attracted a great deal of attention. The optical feeder allows compact and cost-effective base stations, easier channel assignment control, and flexible communication systems. This paper investigates the simultaneous influence of the effects of the wireless channel such as fading and cochannel interference (CCI) and the effects of the optical channel, like shot noise and intermodulation distortion, introduced by the laser diode on the bit error rate (BER) performance of a fiber-fed microcellular system using the $\pi / 4$-differential quadrature phase-shift keying (DQPSK) modem scheme. The wireless channel is assumed to be a frequency-selective, slow, and CCI-limited Nakagami fading channel. The BER performance of the system is studied under various channel conditions using an exact model and a simplified model and a comparative study with a nonfiber-fed system is carried out. The tradeoff between the system capacity and the BER performance in fiber-optic microcellular (FOM) systems is also discussed. The results obtained justify the application of fiber-optic feeders as the remoting infrastructure for future microcellular systems.


Index Terms-Fiberoptic mobile systems, hybrid fiber/wireless systems, Nakagami fading.

## I. INTRODUCTION

Fiber-optic microcellular (FOM) systems have been studied for their application in enhancing the performance of cellular systems [1]-[3]. This paper investigates the simultaneous influence of the effects of the wireless channel like frequency-selective fading and cochannel interference ( CCI ) and the effects of the optical channel such as shot noise and intermodulation distortion (IMD) on the bit error rate (BER) performance of an FOM system using the $\pi / 4$-differential quadrature phase-shift keying (DQPSK) modem scheme. The Nakagami distribution [4] is chosen to model the fading experienced by the desired, delayed, and interfering signals because of its versatility and flexibility in modeling various fading environments. It is assumed that long-term fading is compensated for by power control schemes so that the only fading considered here is short-term fading.
This paper only considers the uplink because the signal encounters the statistical effects of the wireless channel before the optical transmission making the analysis of this uplink more crucial than that of the downlink. Models to evaluate the BER in an FOM system that consider the Gaussian minimum shift keying (GMSK) and binary phase-shift keying (BPSK) modem schemes and a code division multiple access (CDMA)-based system in Rayleigh fading channels are developed in [2] and [3]. To the best of the authors' knowledge, there seems to be little or no work available in the existing literature dealing with the performance of an FOM system using $\pi / 4$-DQPSK in a frequency-selective CCI-limited Nakagami fading scenario. A general expression for the BER of $\pi / 4$-DQPSK in a flat, slow, and CCI-free Nakagami fading channel has been derived in [5]. The primary objective is to incorporate the effects of frequency selectivity, CCI, and laser diode (LD) nonlinearity and noises emanating in the fiber-optic link into this BER model.

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