SEQUENTIAL DETECTOR FOR NONLINEAR CHANNELS WITH APPLICATIONS TO SATELLITE COMMUNICATIONS

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ABSTRACT

Sequential detection over a bandlimited nonlinear channel is considered and a particle filtering algorithm is developed. The nonlinearity of the problem motivates the use of sequential Monte Carlo methods. Since the channel is bandlimited, the resulting memory of the channel allows for modeling the problem as a dynamic state space model. A particular application to nonlinear satellite communication is illustrated, where the channel is a cascade of linear filters and the nonlinear traveling wave tube amplifier at the satellite repeater. The approach results in very simple detectors with good performance characteristics and general applicability.

1. INTRODUCTION

Communication channels are often represented by linear models due to their tractability. However, nonlinearities often persist in many channels of interest, examples of which include satellite [1] and microwave channels [2], and magnetic recording systems [3]. These nonlinearities are often present due to nonlinear devices in the systems, (such as the traveling wave tube (TWT) amplifier [4](and references therein)) and they need to be compensated for effective electronic communication. Approaches that address the nonlinearities include neural networks, Volterra filters, wavelet networks and non parametric techniques using higher order statistics. Identification of nonlinear channels using Volterra series is considered in [1],[5] (see references therein), where the Volterra series are used to model the overall input-output function. In [6] for example, neural networks are applied to model nonlinear channels with memory, where the channel may or may not be represented by separable parts. Optimal sequence detectors consisting of a bank of matched filters followed by maximum likelihood sequence detectors are described in [7]. Recently, a Markov chain Monte Carlo approach has been applied for blind sequence equalization for a satellite channel [8].

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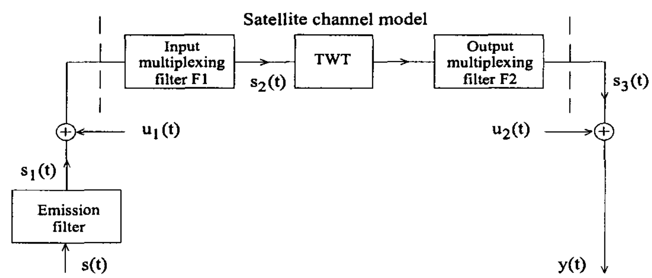

Fig. 1. Baseband model of satellite communication system

In this paper, we present a sequential detector using particle filters for bandlimited nonlinear channels, with applications to satellite systems. Particle filters [9],[10],[11],[12] are becoming popular techniques for a large number of signal processing problems, including those in communications. In [12],[13],[14], a general description of application of particle filters in sequential estimation and detection is given, for linear and nonlinear channels. In this paper, we develop detectors based on particle filters for use in nonlinear satellite channels consisting of a cascade of linear filters and the memoryless nonlinear TWT. Note that due to the presence of linear filters, the channel itself is not memoryless; this allows the dynamic state space (DSS) representation of the system.

Section 2 discusses the satellite system. Particle filtering is described in Section 3. The sequential detection algorithm is proposed in Section 4. Simulations are presented in Section 5 and extensions are provided in Section 6.

2. SATELLITE SYSTEM MODEL

The satellite communication system consists of two earth stations connected by a satellite repeater through two radio links. Data b_k from a discrete complex set $\mathcal{L} = \{l_1, \dots, l_{|\mathcal{L}|}\}$ is transmitted over the baseband bandlimited satellite system described in Figure 1. The symbol period is T and the signal received is sampled at rate $1/T_s$ where $T = pT_s$ and p is the upsampling factor. The advantages of having $p > 2$ are discussed in [8]. The emission filter is a pulse shaping filter and limits the bandwidth of the transmitted signal $s(t) = \sum_k b_k \delta(t - kT)$. The emission filter is a 4-pole Chebyshev filter with 3-dB bandwidth equal to $1.66/T$. The signal $s_1(t)$ is transmitted to the satellite and is affected by additive, zero mean, complex Gaussian noise $u_1(t)$ on the uplink channel. The real and imaginary components are i.i.d. each with variance $\sigma_1^2/2$. The signal received by the satellite has 15dB signal-to-noise ratio(SNR). The input and output multiplexing filters F1 and F2 are also 4-pole Chebyshev filters with 3-dB bandwidth given by $2/T$ and $3.3/T$ respectively. F1 and F2 limit the noise power in the signal. The high power TWT amplifier is a memoryless nonlinearity, with a complex transfer function that depends only on the input amplitude. The amplitude-amplitude (AM/AM) and amplitude-phase (AM/PM) distortions are represented by [4]

$$A(r) = \frac{\alpha_a r}{1+\beta_a r^2}, \quad \phi(r) = \frac{\alpha_p r^2}{1+\beta_p r^2} \tag{1}$$

where r and $A(r)$ are the amplitudes of the input and output signals respectively and $\phi(r)$ is the phase change in the output signal. Values of the parameters $\alpha_a, \beta_a \alpha_p, \beta_p$ are estimated in [4]. The downlink channel has distortion modeled an additive, zero mean, complex Gaussian noise $u_2(t)$ with real and imaginary components i.i.d. with variance $\sigma_2^2/2$. For the sequential detection algorithm presented, we assume that all the system parameters, namely the filter coefficients, satellite parameters and noise variances are known.

Since the signal $y(t)$ at the receiver is upsampled with factor p, denote $a_k = b_{\lceil \frac{k}{p} \rceil}$ where $\lceil \cdot \rceil$ indicates the smallest integer greater than or equal to $(\cdot)$. The various discrete signals at each stage of the system are represented below. We use the notation

$$p_k = F(\mathbf{c}, \mathbf{d}, q_k) \tag{2}$$

to indicate the transversal linear filtering operation

$$p_k = -c_1 p_{k-1} - \ldots - c_{m_1} p_{k-m_1} + d_0 q_k + \ldots + d_{m_2} q_{k-m_2} \tag{3}$$

where q_k and p_k are the input and output of the filter respectively with coefficients $\mathbf{c} \equiv (c_1, \ldots, c_{m_1})$, $\mathbf{d} \equiv (d_1, \ldots, d_{m_2})$.

We have the following DSS model for the satellite communication system,

$$\begin{aligned} s_{1,k} &= F(\mathbf{c_0}, \mathbf{d_0}, a_k) \\ s_{2,k} &= F(\mathbf{c_1}, \mathbf{d_1}, s_{1,k} + u_{1,k}) \\ s_{3,k} &= F(\mathbf{c_2}, \mathbf{d_2}, TWT(s_{2,k})) \\ y_k &= s_{3,k} + u_{2,k} \end{aligned} \tag{4}$$

where $TWT(\cdot)$ is the TWT amplifier transfer function described by (1) and all the filters are 4-pole Chebyshev.

3. PARTICLE FILTERS

This section explains particle filters, where we employ a change of notation. Consider the following DSS model,

$$\begin{aligned} x_k &= f_k(x_{k-1}, u_k) & \text{(state equation)} \\ y_k &= h_k(x_k, v_k) & \text{(observation equation)} \end{aligned} \tag{5}$$

where x_k, y_k, u_k and v_k are the hidden state, observation, process noise and observation noise respectively, of given dimensionality. From a Bayesian point of view, we would like to estimate $p(x_k|\mathbf{y}_k)$, where $\mathbf{y}_k^T = (y_1, \ y_2, \ldots, y_k)$. Also of interest is the expectation $E_p(g(x_k)|\mathbf{y}_k)$. For a linear model with Gaussian noise, $p(x_k|\mathbf{y}_k)$ is Gaussian and the celebrated Kalman filter can be sequentially used to obtain a closed form solution. However, with nonlinearity (as in our problem) and non-Gaussianity in the model, there generally exist no such closed form solutions and analytical computation is infeasible practically. Monte Carlo based filters provide a practical methodology for estimation in such problems.

The basic idea is to represent the distribution as a collection of samples (particles) from that distribution. N particles, $\mathcal{X}_k = \{x_k^{(1)}, \ldots, x_k^{(N)}\}$, from the so called importance sampling (IS) distribution $\pi(x_k|\mathbf{y}_k)$ are generated. Subsequently, the particles are weighted as $w_k^{(i)} = \frac{p(x_k^{(i)}|\mathbf{y}_k)}{\pi(x_k^{(i)}|\mathbf{y}_k)}$. Let $W_k = \{w_k^{(1)}, \ldots, w_k^{(N)}\}$, then the set $\{\mathcal{X}_k, W_k\}$ represents samples from the posterior distribution $p(x_k|\mathbf{y}_k)$. An estimate of $E_p(g(x_k))$ can be written as:

$$\hat{E}_p(g(x_k)) = \sum_i w_k^{(i)} g(x_k^{(i)}). \tag{6}$$

Due to the Markovian nature of the state equation, we can obtain a sequential procedure called sequential importance sampling (SIS), to obtain estimates of $p(x_k|\mathbf{y}_k)$ sequentially [9],[10]. The algorithm can be written as follows:

1. At time $k = 0$, we start with N samples from $\pi(x_0|\mathbf{y}_0)$ and denote them $x_0^{(i)}; n = 1, \ldots, N$, with weights $w_0^{(i)} = p(x_0^{(i)}|\mathbf{y}_0)/\pi(x_0^{(i)}|\mathbf{y}_0)$.
2. At time $k = 1, \ldots, K$, let $\mathcal{X}_k = \{\mathbf{x}_k^{(i)}; i = 1, \ldots, N\}$ be samples with weights $W_k = \{\tilde{w}_k^{(i)}; i = 1, \ldots, N\}$. The sets $\mathcal{X}_{k-1}$ and W_{k-1} represent the posterior density $p(x_{k-1}|\mathbf{y}_{k-1})$. We obtain particles and weights for time k from steps 3, 4 and 5:
3. For $i = 1, \ldots, N$, sample $x_k^{(i)} \sim \pi(x_k|x_{k-1}, \mathbf{y}_k)$.
4. For $i = 1, \ldots, N$, update the weights using:

$$w_k^{(i)} = \tilde{w}_{k-1}^{(i)} \frac{p(y_k|x_k^{(i)}) p(x_k^{(i)}|x_{k-1}^{(i)})}{\pi(x_k^{(i)}|x_{k-1}^{(i)}, \mathbf{y}_k)}. \tag{7}$$

5. Normalize the weights using $\tilde{w}_k^{(i)} = w_k^{(i)} / \sum_{j=1}^{N} w_k^{(j)}$.

In SIS, degeneration of particles occurs with time k. In effect, the weights of only a few particles remain significant. This results in a poor estimate of the expectation in (6). A procedure called *resampling* can be used to reduce this degeneration. The basic idea is to duplicate the particles which have significant weights, in proportion to the weights of the particles. For a detailed description, see for example [10].

4. SEQUENTIAL NONLINEAR DETECTOR

From the DSS model (4), we observe that the hidden (unknown) states are the filter outputs $s_{1,k}, s_{2,k}, s_{3,k}$ and the transmitted data $a_k (\equiv b_k)$. Clearly due to the nonlinear TWT amplifier, this DSS model is nonlinear in the state equation which motivates the application of particle filters for sequential detection. However, note that $s_{1,k}, s_{2,k}, s_{3,k}$ are nuisance variables and our primary interest is in the detecting b_k. Also, $s_{1,k}$ and $s_{3,k}$ are deterministic functions of b_k and $s_{2,k}$.

In order to use SIS algorithm, we need to choose the IS function $\pi(\cdot)$. Following the above observations, we need only generate particles for b_k and $s_{2,k}$. Subsequently, particles of $s_{1,k}$ and $s_{3,k}$ can be obtained from the deterministic equations in (4). Due to the nonlinearity of state equation and non-additive noise, a simple choice is the *prior* distribution

$$\begin{aligned} \pi \equiv \; & p(b_k, s_{2,k} | b_{k-1}, s_{2,k-1:k-4}) \\ & = p(b_k) p(s_{2,k} | b_{k:k-1}, s_{2,k-1:k-4}) \end{aligned} \quad (8)$$

where the last equality comes from assuming i.i.d. transmitted data (this assumption can be relaxed for say encoded bits, see for example [14]). The SIS algorithm for sequential detection is given below.

4.1. SIS Algorithm

For $j = 1, \ldots, Kp$; $k = 1, \ldots, K$, repeat the following.

1. For $i = 1, \ldots, N$, generate $b_j^{(i)} \sim p(b_j)$, where $p(b_j)$ is known *a priori*.

2. Since a_k is an unsampled version of b_j, we have $a_k^{(i)} = b_j^{(i)}$ where $j = \lceil \frac{k}{p} \rceil$, $i = 1, \ldots, N$.

3. For $i = 1, \ldots, N$, obtain $s_{1,k}^{(i)} = F(\mathbf{c_0}, \mathbf{d_0}, a_k^{(i)})$.

4. For $i = 1, \ldots, N$, generate samples $u_{1,k}^{(i)}$ from its Gaussian distribution.

5. For $i = 1, \ldots, N$, obtain $s_{2,k}^{(i)} = F(\mathbf{c_1}, \mathbf{d_1}, s_{1,k}^{(i)} + u_{1,k}^{(i)})$. and compute $s_{3,k}^{(i)} = F(\mathbf{c_2}, \mathbf{d_2}, TWT(s_{2,k}^{(i)}))$.

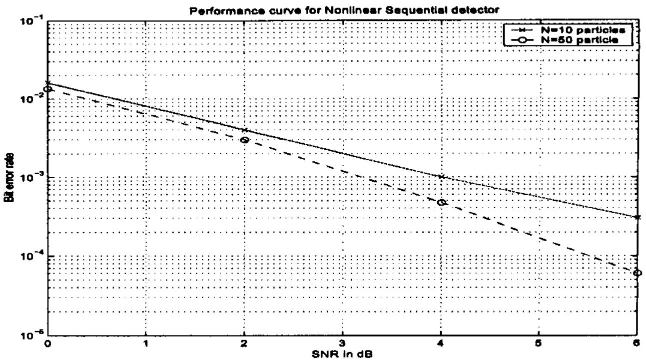

Fig. 2. Plot of SNR Vs. BER characteristics of proposed Sequential nonlinear detector

6. Update weights using

$$w_k^{(i)} = w_k(i)\, p(y_k | s_{3,k}^{(i)}), \quad i = 1, \ldots, N. \quad (9)$$

7. Normalize the weights using $\bar{w}_k^{(i)} = w_k^{(i)} / \sum_{i=1}^{N} w_k^{(i)}$.

8. Resample the particle trajectories according to a resampling schedule [10].

4.2. Detection

Using the particles obtained in the above algorithm, decisions on the received data are made in the following manner. Since there are p observations per symbol period, we must use all the p observations to make decisions about the transmitted symbol. Evidently, decisions on the transmitted bits are made at times $k = p, 2p, \ldots, Kp$. The posterior probability of b_k can be written as:

$$\begin{aligned} P(b_k = l_m | \mathbf{y}_k) &= E(\mathbf{I}(b_k = l_m)) \\ &\approx \sum_{i=1}^{N} \bar{w}_k^{(i)} \mathbf{I}(b_k^{(i)} = l_m) \qquad m = 1, \ldots, |\mathcal{L}|, \end{aligned} \quad (10)$$

where $\mathbf{I}(b_k = l_m) = 1$ if $b_k = l_m$ and 0 otherwise. Then choose $b_k = l_i$ so that,

$$\hat{b}_k = \max_{l_m \in \mathcal{L}} P(b_k = l_m). \quad (11)$$

This decision process does indeed take into account all the received data. The weights at time say $k = p$ are smoothed weights for the decision on a_1 (at time $k = 1$), i.e. w_p takes into account the observations $y_{1:p}$ for the decision at time $k = 1$. This is nothing but fixed-lag smoothing as discussed for example in [14].

5. SIMULATIONS AND COMMENTS

In the simulations, the BPSK constellation was considered, so that $\mathcal{L} = \{+1, -1\}$. Data are transmitted at rate 1 Mbaud, i.e. $T = 10^{-6}s$. The upsampling factor was $p = 7$. The coefficients for the Chebyshev filters were obtained using the cheby2.m function in MATLAB. Values for the satellite parameters were chosen from [4], $\alpha_a = 2.1587, \beta_a = 1.1517, \alpha_p = 4.0033$, and $\beta_p = 9.1040$. The uplink SNR was maintained at 15dB. A number of simulations were carried out and the results shown are averaged over 10 random Monte Carlo runs for each SNR. Figure (2) shows the bit error rate (BER) versus SNR characteristics of the proposed detector which used $N = 10$ and $N = 50$ particles. Observe that increasing the number of particles leads to an improvement in the performance. It is interesting to note that even for small number of particles $N = 10$, the performance is good. This is because the uplink SNR is maintained at 15dB (even in practical systems) and the filters merely limit the noise power in the received signal while adding a memory of 4 to the system.

6. EXTENSIONS AND CONCLUSIONS

A particle filtering detector was developed for a bandlimited nonlinear channel and its application was illustrated for a satellite communications system. Some extensions are also possible. First note that the detection algorithm also applies for non-Gaussian channel noises, since the weight update function depends only on the distribution of the channel noise, which is a very important property. Secondly, the signal received at the receiver can also suffer from multipath fading on the downlink. This too can be incorporated using techniques from [13], where the fading channel is modeled as an autoregressive process. Thirdly, the satellite parameters can also be included in the unknown state variable. This was treated in [8], where the parameters came from a discrete set. Finally, nonlinear decoding can be treated in a similar fashion. Since encoded bits are not i.i.d., samples of b_k can be generated from $p(b_k|b_{k-1:k-m})$ which is established from the encoding scheme.

The proposed detector can be extended to many communication channels. It has the added advantage of being parallelizable and tractable for VLSI implementation.

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