MULTIUSER DETECTION OF SYNCHRONOUS CDMA SIGNALS BY THE GIBBS COUPLER

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ABSTRACT

Code-division multiple-access (CDMA) is a multiplexing technique which has become a driving force behind the rapidly advancing communications industry. In order to recover transmitted signals at the receiver when CDMA is used, multiuser detection techniques are engaged. In the past, various techniques have been developed to approach the performance of the optimum multiuser detector, and in this paper we have the same objective. We propose a new multiuser detector which is developed under the Bayesian framework and implemented by a novel efficient perfect sampling algorithm called the Gibbs coupler. The simulation results demonstrate excellent performance of the proposed detector.

1. INTRODUCTION

Code-division multiple-access (CDMA) is a multiplexing technique which enables multiple users to access a common channel simultaneously [1]. CDMA systems demonstrate significant advantages over analog and conventional time-division multiple access (TDMA) systems such as increased capacity and reduced effects of multipath fading. As a result, the CDMA technology has become a driving force behind the rapidly advancing communications industry.

In order to recover the transmitted signals at the receiver, multiuser detection techniques are employed. Multiuser detection techniques overcome the disadvantage of the single-user matched filter, in which no information about the actual interference among users is exploited. In the past, different techniques have been developed to approach the performance of the optimum detector. Among them, the class of linear detectors including the decorrelating detector is very popular and well studied, primarily due to its computational simplicity. However, in many cases the performance of the linear detectors is far from optimal, and therefore better solutions have been proposed. Among them are methods based on Markov chain Monte Carlo (MCMC) sampling [2].

We present here a new multiuser detector which is developed under the Bayesian framework and realized by the Gibbs coupler [3]. The Gibbs coupler belongs to a class of sampling based methods called perfect sampling algorithms. Perfect sampling was motivated by the study of convergence analysis of MCMC and was initiated by Propp and Wilson [4]. In contrast to MCMC, perfect sampling obtains i.i.d samples exactly from desired distributions, which completely complies with the requirement of the Monte Carlo method. Hence, statistical methods based on perfect sampling achieve better performance than methods based on MCMC sampling.

In the sequel, the problem of optimum multiuser detection is stated first. This is followed by a review of the basic concept of perfect sampling and introduction of the Gibbs coupler on high dimensional binary state spaces. Then, multiuser detection by the Gibbs coupler is proposed. Finally, simulation results are provided that show the performance of the proposed detector.

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2. OPTIMUM MULTIUSER DETECTION

A K-user synchronous CDMA white Gaussian channel can be modeled as

$$y(t) = \sum_{k=1}^{K} A_k b_k s_k(t) + n(t), \quad t \in [0, T], \tag{1}$$

where $y(t)$ denotes the received signal, $s_k(t)$ is the antipodal signature waveform of the k-th user, A_k is the amplitude of the k-th user's signal, $b_k \in \{-1, 1\}$ is the bit transmitted by the k-th user, $n(t)$ represents additive white Gaussian noise with zero mean and variance σ^2, and T denotes the symbol duration. We assume here that all the parameters except the b_k's are known, and our objective is to estimate $\mathbf{b}^\top = [b_1 b_2 \cdots b_K]$. The conventional single-user matched filter makes the decision on each user separately and the estimate of the k-th user can be expressed as

$$\hat{b}_k = sgn(y_k) \tag{2}$$

where $y_k = \int_0^T s_k(t) y(t) dt$ is the k-th matched filter output. It is noticed that the single-user matched filter does not use information about the users' correlations. Therefore it is not optimal in the presence of multiuser interference which exists in CDMA systems. To achieve optimum detection, one must employ a multiuser detection strategy. From a Bayesian perspective, the optimum decision is made from the posterior distribution of $\mathbf{b}$. Since *a priori* nothing is known about $\mathbf{b}$, noninformative prior is chosen for $\mathbf{b}$. It thus follows that the posterior distribution of $\mathbf{b}$ can be expressed as

$$\begin{aligned} p(\mathbf{b}|y(t), t \in [0,T]) \\ &\propto \exp\left(-\frac{1}{2\sigma^2}\int_0^T [y(t) - \sum_{k=1}^{K} A_k b_k s_k(t)]^2 dt\right) \\ &\propto \exp\left(\frac{1}{2\sigma^2}(2\sum_{k=1}^{K} A_k y_k b_k - \sum_{k=1}^{K}\sum_{l=1}^{K} A_k A_l \rho_{kl} b_k b_l)\right) \end{aligned} \quad (3)$$

where $\rho_{kl} = \int_0^T s_k(t)s_l(t)dt$ represents the crosscorrelation between the k-th and the l-th signature waveform. Then the most likely $\mathbf{b}$ is the set that maximizes the posterior distribution (3). The solution leads to the *maximum a posteriori* (MAP) detector. In the following sections, we show how a new perfect sampling algorithm called the Gibbs coupler can be applied to calculate the output of the MAP detector.

3. THE GIBBS COUPLER

The Gibbs coupler [3] is a novel perfect sampling algorithm. By perfect sampling we refer to a class of sampling based algorithms that are able to draw exact i.i.d. samples from a desired distribution by running Markov chains. The initial perfect sampling algorithm is called *coupling from the past* (CFTP) [4], and it was designed on discrete state spaces. To illustrate the basic idea of CFTP, let us assume that the discrete sampling space $\mathcal{S}$ is of size $M = |\mathcal{S}|$. Then, CFTP constructs M Markov chains at every possible state in $\mathcal{S}$ and runs them from the infinite past to time 0. It is noted that all the Markov Chains should have the desired distribution as their stationary distribution, and at any instant of time t in the past the same random seed $R^{(t)}$ and updating function $\Phi(\cdot, R^{(t)})$ are applied to every chain in order to determine their new states. Now, if there comes a time when all the chains have reached the same state, from this time onward all the chains follow the same path due to the common random seed and updating function. This means that the effect of the initial states on the coalesced state has actually worn off. Apparently, since chains have lasted infinitely long from the past, the coalesced state at time 0 is a steady state. Then at time 0, this coalesced state is a perfect sample from the desired distribution.

The implementation of CFTP on large state spaces is rather complex due to the heavy computational burden in tracing all the Markov chains started from every possible state. Computationally efficient algorithms have been proposed [4, 5] for problems where *monotonic* or *antimonotonic* Markov chains exists. In these cases, a sandwiched CFTP can be employed where only chains started at two extreme states are traced. For more details on perfect sampling, see [4, 6].

In our recent work [3], we have proposed a perfect sampling algorithm termed the Gibbs coupler. The Gibbs coupler combines the features of CFTP and the Gibbs sampler. Its importance is stressed when sampling from high dimensional state spaces is needed. A version of the algorithm on the multi-dimensional binary variable space demonstrates that the Gibbs coupler is able to handle general cases where (anti-)monotonicity does not exist.

Now, we briefly summarize the Gibbs coupler on the multi-dimensional binary variable space. Let $\boldsymbol{\mathcal{S}}^{(t)} = \{\mathcal{S}_1^{(t)}, \mathcal{S}_2^{(t)}, \cdots, \mathcal{S}_N^{(t)}\} \in \{-1,1\}^N$ denote the support of $\mathbf{x}$ at time t where $\mathcal{S}_i^{(t)}$, $i = 1, 2, \cdots, N$ represents the support of the component x_i at time t. Then the basic Gibbs coupler that draws exact samples from $p(\mathbf{x})$ is given by the following pseudo-code:

```
Gibbs coupler(T):
t ← −T
while t < 0
    t ← t + 1
    i ← 0
    while i <= N
      update S_i^(t) ~ p(x_i | S_1^(t), ..., S_{i-1}^(t), S_{i+1}^(t-1), ..., S_N^(t-1))

if size of all S_i^(0) for i = 1, 2, ··· N is equal to 1 then
    return(S^(0))
else
    Gibbs coupler(2T)
```

The overall framework of the algorithm still follows that of CFTP. This framework actually guarantees that unbiased samples from the desired distribution are obtained if coalescence occurs. However, in contrast to the CFTP algorithm, the main coupling method of the Gibbs coupler is component based. In particular, at any time instant t, sandwich distributions are required for $i = 1, 2, \cdots, N$, which are defined as

$$L_i^{(t)}(x_i = 1) \quad = \quad \min_{\mathbf{x}_{-i}^{(t)} \in \boldsymbol{\mathcal{S}}_{-i}^{(t)}} \left\{p(x_i = 1|\mathbf{x}_{-i}^{(t)})\right\} \quad (4)$$

and

$$U_i^{(t)}(x_i = 1) \quad = \quad \max_{\mathbf{x}_{-i}^{(t)} \in \boldsymbol{\mathcal{S}}_{-i}^{(t)}} \left\{p(x_i = 1|\mathbf{x}_{-i}^{(t)})\right\} \quad (5)$$

where $\boldsymbol{\mathcal{S}}_{-i}^{(t)} = \{\mathcal{S}_1^{(t)}, \cdots, \mathcal{S}_{i-1}^{(t)}, \mathcal{S}_{i+1}^{(t-1)}, \cdots, \mathcal{S}_N^{(t-1)}\}$ denotes the collection of supports of $\mathbf{x}_{-i}$ at time t with the individual component supports at time t being $\{-1\}$, $\{1\}$, or $\{-1, 1\}$. The update of the support $\mathcal{S}_i^{(t)}$ is then formulated as

$$\mathcal{S}_i^{(t)} = \Phi(\boldsymbol{\mathcal{S}}_{-i}^{(t)}, R_i^{(t)}) = \begin{cases} \{1\}, & \text{if } R_i^{(t)} \leq L_i^{(t)}(x_i = 1) \\ \{-1\}, & \text{if } R_i^{(t)} \geq U_i^{(t)}(x_i = 1) \\ \{-1, 1\}, & \text{otherwise} \end{cases} \quad (6)$$

where $R_i^{(t)}$ is a uniform random seed.

In [3] it is proved that the sandwich distributions chosen according to (4) and (5) achieve the largest probability of coalescense, and hence their use leads to the fastest coalescense. In [3], several properties of the Gibbs coupler have been shown. The properties lead to two important theorems. First, the Gibbs coupler has the same rate of coalescense as CFTP, but it is computationally much more efficient especially for high dimensional state spaces. Secondly, for the problems where monotonicity exists, it is equivalent to the sandwiched CFTP algorithm. For more details, see [3].

4. MULTIUSER DETECTION BY THE GIBBS COUPLER

In this section, we show how the Gibbs coupler can be applied to tackle the multiuser detection problem. From the discussion in the last section, we know that an important issue in the implementation of the Gibbs coupler is the determination of the sandwich distributions on the full conditional distributions. For the problem of multiuser detection, the full conditional distributions are readily derived from the posterior distribution (3) as

$$\begin{aligned} p(b_i = 1|\mathbf{b}_{-i}, y(t), t \in [0,T]) \\ \propto \exp\left(\frac{1}{2\sigma^2}(2A_i y_i - 2\sum_{k=1,k\neq i}^{K} A_i A_k \rho_{ki} b_k)\right) \\ = \left[1 + \exp(\frac{2}{\sigma^2}(-A_i y_i + \sum_{k=1,k\neq i}^{K} A_i A_k \rho_{ki} b_k))\right]^{-1} \end{aligned} \quad (7)$$

for $i = 1, 2, \cdots K$. We notice that the maximum and the minimum on (7) with respect to $\mathbf{b}_{-i}$ can be easily determined only by checking the sign of $A_i A_k \rho_{ki}$. It thus follows that, at time t, the sandwich distributions defined by (4) and (5) are

$$\begin{aligned} L_i^{(t)}(b_i = 1) \\ = \left[1 + \exp(\frac{2}{\sigma^2}(-A_i y_i + \sum_{k\in \mathbf{I}_{i1}^{(t)}} |\beta_k| + \sum_{k\in \mathbf{I}_{i2}^{(t)}} \beta_k b_k^{(t)}))\right]^{-1} \end{aligned} \quad (8)$$

and

$$\begin{aligned} U_i^{(t)}(b_i = 1) \\ = \left[1 + \exp(\frac{2}{\sigma^2}(-A_i y_i - \sum_{k\in \mathbf{I}_{i1}^{(t)}} |\beta_i| - \sum_{k\in \mathbf{I}_{i2}^{(t)}} \beta_k b_k^{(t)}))\right]^{-1} \end{aligned} \quad (9)$$

where $\beta_k = A_i A_k \rho_{ki}$, $\mathbf{I}_{i1}^{(t)} \subset \{1, 2, \cdots, i-1, i+1, \cdots, K\}$ contains the indices of the elements of $\{b_k^{(t)}\}_{k=1,k\neq i}^{K}$ that have not coalesced at time t, and $\mathbf{I}_{i2}^{(t)} \subset \{1, 2, \cdots, i-1, i+1, \cdots, K\}$ are the indices of the remaining elements of the set $\{b_k^{(t)}\}_{k=1,k\neq i}^{K}$ that have coalesced at time t. Then according to the algorithm, at any time t, the support of the i-th component is updated according to the function (6), and the coalesced state at time $t = 0$ is recorded as a perfect sample from the posterior (3). Suppose that the desired size of the samples is N. Once N samples are acquired, the MAP estimate is computed. There are several approaches to find the MAP estimate. For instance, the posterior probability of each perfect sample is computed, and the MAP estimate is the one which has the largest posterior probability. As an alternative, one can consider the samples of each b_i, for $i = 1, 2, \cdots, N$ independently. In this case, the MAP estimate of b_i is set to be the sample that appears most frequently. This procedure yields the marginalized MAP estimate of the transmitted bits.

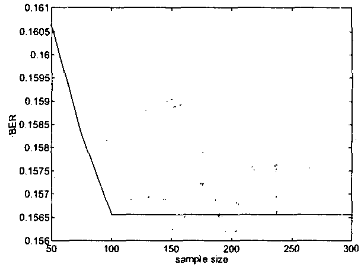

Figure 1: BER of the MAP detector implemented by the Gibbs coupler as a function of the sample size N. There were 5 users with equal power, and SNR = 0 dB.

5. SIMULATION RESULTS

In this section we present the results of numerical simulations that were carried out to demonstrate the performance of the MAP detectors by the Gibbs coupler. We conducted several experiments. In the first three experiments, a 7-bit Gold sequence was used as the spreading code of a 5-user system. In the first experiment, the users were assumed with equal power. Bit error rates (BERs) of the MAP detector implemented by the Gibbs coupler were computed on 3000 transmitted bits for different sample sizes N. The signal-to-noise ratio (SNR) was fixed at 0 dB. The results for the first user are plotted in Figure 1. It shows that the BER does not change when the sample size N is greater than 100. It indicates that $N = 100$ is sufficient to obtain reliable performance, and therefore N was set to 100 in the following two experiments.

In the second experiment, BERs of both the MAP and the marginalized MAP detectors of the first user were examined under different SNRs. The results are illustrated in Figure 2. The results of the conventional matched filter and the decorrelating detector on the same user are also presented. The theoretically attainable performance in the absence of multiuser interference is plotted as a lower bound. In particular, in order to compute the BER at a specific SNR for any detector, Monte Carlo trials were performed. The number of Monte Carlo trials was pre-computed by assuring that, under the BER of the lower bound at this SNR, there would be at least 300 errors among the trials. We see that the curves corresponding to the MAP and the marginalized MAP detectors overlap with each other, which indicates similar performance of these detectors. In addition, the BERs of the two MAP detectors are very close to the bound whereas neither the single-user matched filter nor the decorrelating detector is able to achieve the same performance.

In the third experiment, we simulated the situation of near-far effect. Different energy levels were assigned to the five users. The two largest energy levels were 5dB higher and the two middle energy levels were 3 dB higher than the smallest energy level, respectively. BERs of the MAP and the marginalized MAP detectors of the user with the smallest energy are plotted versus SNR in Figure 3. Their performance was also compared with that of the matched filter and the decorrelating detector. It demonstrates that the

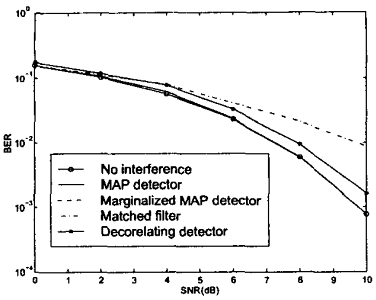

Figure 2: BER of the different methods on a 5-user-equal-power system as a function of SNR.

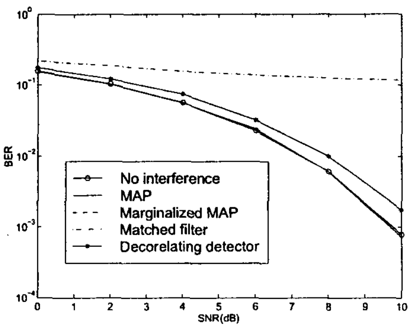

Figure 3: BER of the user with smallest amplitude level as a function of SNR. There were 5 users with different amplitude levels.

two MAP detectors have smaller BERs than the matched filter and the decorrelating detector. Moreover, they also approach the lower bound which is plotted as the theoretical optimal BER in the absence of multiuser interference.

Next, in the forth experiment, a higher capacity system of 15 users was introduced. We assumed that all users were with equal power. First in order to determine the sample size, a similar test as in the first experiment was conducted. The BERs at 0 dB is plotted in Figure 4 as a function of the sample size. Obviously, 200 samples are sufficient for obtaining reliable results.

Once the sample size was determined, the BERs of the MAP detectors, the matched filter, and the decorrelating detector were measured on the system and the results are depicted in Figure 5. We can see that the MAP detectors have a similar performance, which is very close to the lower bound. In addition, they clearly outperform the single-user matched filter and the decorrelating detector.

6. CONCLUSION

We successfully employed the Gibbs coupler to compute optimal MAP detection of synchronous CDMA signals. The numerical simulation results on users with equal powers showed that the proposed MAP detectors have better performance than either the single-user matched filter or the decorrelating detector. The simulation results of near-far effect scenarios, too, showed better performance of the proposed MAP detectors.

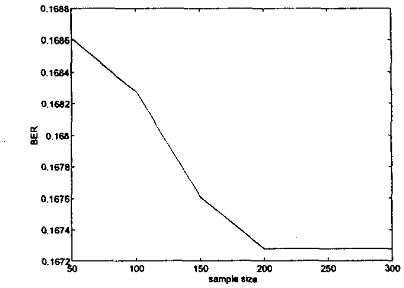

Figure 4: BER of the MAP detector on a 15-user-equal-power system as a function of N and for SNR = 0 dB.

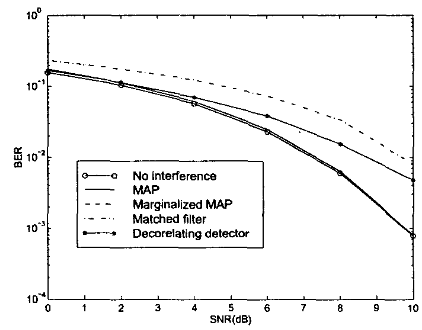

Figure 5: BER of the different methods on a 15-user-equal-power system as a function of SNR.

7. REFERENCES

[1] S. Verdú, *Multiuser Detection*, Cambridge University Press, New York, 1998.

[2] X. Wang and R. Chen, "Adaptive Bayesian multiuser detection for synchronous CDMA with Gaussian and impulsive noise," *IEEE Trans. on Signal Processing*, vol. 47, no. 7, pp. 2013–2027, July 2000.

[3] Y. Huang and P. M. Djurić, "The Gibbs coupler: A novel perfect sampling algorithm," *submitted to IEEE Trans. on Information Theory*, 2000.

[4] J. G. Propp and D. B. Wilson, "Exact sampling with coupled Markov chains and applications to statistical mechanics," *Random Structures Algorithms*, vol. 9, pp. 223–252, 1996.

[5] O. Haggstrom and K. Nelander, "Exact sampling from antimonotone system," in *Research Rep. 1997-03*, Department of Mathematics, Chalmers University of Technology and Goteborg University, 1997.

[6] Elke Thönnes, "A primer on perfect simulation," *available at http://dimacs.rutgers.edu/~dbwilson/exact*, 2000.