Ballistic versus diffusive base transport in the high-frequency characteristics of bipolar transistors

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(Received 21 November 1991; accepted for publication 16 March 1992)

The time-dependent Boltzmann equation is used to calculate the small-signal complex base transport factor $\alpha(\omega)$ for different ratios between the base width W and the scattering mean-free path l_{sc}. It is shown that the phase trajectory (Re α, Im α) has a universal character both in the diffusion limit ($W \gg l_{sc}$) and the ballistic limit ($l_{sc} \gg W$). In the latter limit, the trajectory is completely determined by the distribution function of minority carriers injected into the base. The complex trajectories are plotted for several model distributions, including the usual thermal distribution and taking into account the injection energy appropriate for a heterojunction bipolar transistor with a wide-gap emitter.

The purpose of this letter is to identify distinctive manifestations of the collisionless transport of minority carriers across the base of a bipolar transistor. We use the time-dependent Boltzmann transport equation (BTE) to calculate the small-signal complex parameter $\alpha(\omega) \equiv \delta I_C/\delta I_E$. Minority carriers in the base are treated as neutral particles, which is reasonable if the dielectric relaxation time of majority carriers is shorter than ω^{-1}. For a quantitative comparison with experiment, both the capacitive and the transit-time corrections in the emitter-base (EB) and base-collector (BC) junctions must be included separately.

We assume that the base layer is homogeneous in the scattering parameters and that the electron distribution is inhomogeneous only in the z direction. For a periodic perturbation with an angular frequency ω we seek a distribution function in the form

$$f(\mathbf{r},\mathbf{k},t) = f(z,k,u)e^{i\omega t}, \tag{1}$$

where u is the cosine of the angle between $\mathbf{k}$ and the z axis. Parameterizing the collision integrals with a scattering length $l_{sc}(k)$, the BTE for $f(z,k,u)$ can be written in the form[1]

$$u\frac{df(z,k,u)}{dz} + \frac{f(z,k,u)}{l^*(k,\omega)} = \frac{f_0(z,k)}{l_{sc}(k)}, \tag{2}$$

where f_0 is the symmetric (angle-averaged) part of the distribution function,

$$\frac{1}{l^*(k,\omega)} \equiv \frac{i\omega m}{\hbar k} + \frac{1}{l_{tot}(k)}, \quad \frac{1}{l_{tot}} \equiv \frac{1}{l_{sc}(k)} + \frac{1}{l_{cp}(k)}, \tag{3}$$

and $l_{cp}(k)$ is a characteristic length associated with the capture processes. With a perfect sink condition $f(W,k,u<0)=0$ at the BC interface ($z=W$), the integro-differential equation (2) can be reduced to a simple integral equation,

$$f_0(\zeta,k) = \frac{1}{2}\int_0^1 f(0,k,u)e^{-\zeta/u}du + \frac{l^*}{2l_{sc}}\int_0^w f_0(\zeta',k)E_1(|\zeta-\zeta'|)d\zeta', \tag{4}$$

where $\zeta \equiv z/l^*$ and $w \equiv W/l^*$. Equation (4) determines $f_0(\zeta,k)$ in terms of the *in-bound* part $f(0,k,u>0)$ of the distribution function at the EB boundary. The current density $J(z)$ and the complex transport factor $\alpha(\omega)$ are then calculated from

$$G(\zeta,k) = \frac{-ek^2}{2\pi^2\hbar}\left[\int_0^1 f(0,k,u)e^{-\zeta/u}u\,du + \frac{l^*}{l_{sc}}\int_0^w f_0(\zeta',k)E_2(|\zeta-\zeta'|) \times \operatorname{sign}(\zeta-\zeta')d\zeta'\right], \tag{5}$$

$$J(z) = \int_0^\infty G(z,k)dE_k; \quad E_k \equiv \frac{\hbar^2k^2}{2m}, \tag{6}$$

$$\alpha(\omega) = \frac{J(W)}{J(0)}. \tag{7}$$

Functions $E_n(x)$ in Eqs. (4) and (5) are exponential integrals,[2,3] $E_n(x) = \int_0^1 t^{n-2}e^{-x/t}dt$.

In the limit $W \gg l_{sc}$, Eq. (2) reduces to the usual diffusion equation, $i\omega n = D\partial^2 n/\partial z^2 - n/\tau_{cp}$, where $\tau_{cp} = (\pi m/8kT)^{1/2} l_{cp}$. For an energy-independent l_{sc}, the diffusivity D is of the form, $D = (8kT/9\pi m)^{1/2} l_{sc}$. The solution in the diffusion limit is well known:

$$\alpha_d = 1/\cosh\left[\frac{W^2}{D\tau_{cp}} + i\frac{\omega W^2}{D}\right]^{1/2} \underset{\omega\tau_{cp}\gg 1}{\approx} 2e^{-\gamma_d(1+i)},$$
$$\gamma_d \equiv \left[\frac{\omega W^2}{2D}\right]^{1/2}. \tag{8}$$

At sufficiently high frequencies, $\omega\tau_{cp} \gg 1$, the phase of α_d is given by $\arg(\alpha_d) = -\gamma_d$. At the same time, γ_d describes an exponential decrease of the absolute transfer ratio $|\alpha_d|$. The origin of this effect is obvious: the fraction of minority carriers, injected into the base during a half-period π/ω, that returns to the emitter in the subsequent half-period rather than reaches the BC junction, increases with increasing diffusion time W^2/D. As γ_d is varied, α_d traces a universal curve (the logarithmic spiral) in polar coordinates, cf. Fig. 1(a). In the opposite ("ballistic") limit $l_{sc} \gg W$, the second term in Eqs. (4) and (5) is negligible

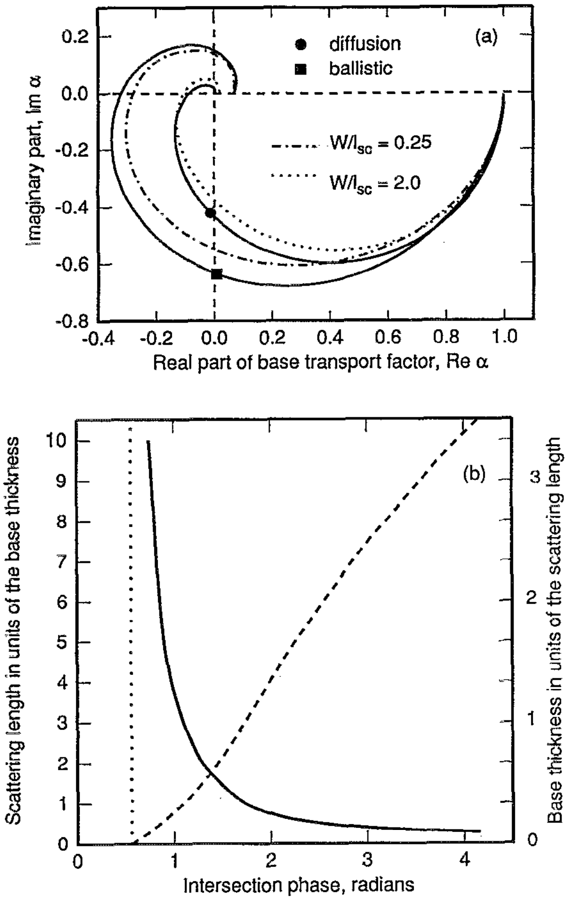

FIG. 1. Phase trajectories for homojunction bipolar transistors. (a) The base transport factor α is traced (e.g., by varying the frequency ω) in the complex plane for different scattering lengths and base thicknesses. Solid curves represent the universal spirals obtained in the diffusion ($l_{sc} \ll W$) and the ballistic ($W \ll l_{sc}$) limits. (b) Dependence of the scattering length l_{sc}/W in a transistor base on the phase γ^x of the intersection point between its α trajectory with the universal spiral corresponding to diffusive transport. Dashed line indicates the corresponding dependence of the dimensionless base thickness W/l_{sc}.

for all reasonable frequencies and $\alpha(\omega)=\alpha_b[f(0,k,u)]$ becomes a functional of the in-bound distribution function. It is evaluated below for several distributions of practical interest.

For homojunction bipolar transistors, the appropriate in-bound distribution[1] is a Maxwellian function $f(0,k,u) \sim e^{-E_k}/kT$. Equations (4)–(7) in this case yield (for $l_{sc}^{-1} \to 0$)

$$\alpha_b(\omega)=2\int_0^\infty E_3\left[i\sqrt{\frac{3}{2}}\frac{\gamma_b}{\sqrt{\epsilon}}+\frac{W}{l_{cp}}\right]\epsilon e^{-\epsilon}\,d\epsilon,\quad \gamma_b\equiv\frac{\omega W}{v_T},\tag{9}$$

where $v_T\equiv(3kT/m)^{1/2}$ is the thermal velocity in a Maxwellian ensemble. For $\omega\tau_{cp}\gg 1$ one has $W\ll\gamma_b l_{cp}$ and the second term in the argument of E_3 can be neglected. Therefore α_b, given by Eq. (9), traces another universal spiral. The curves given by Eqs. (8) and (9) are shown in Fig. 1(a) by the solid lines (assuming $l_{cp}=\infty$). Here and below the spirals are displayed only up to $|\arg(\alpha)|<2\pi$.

If the scattering length is comparable to the base width, then neither the diffusion nor the ballistic approximations are valid. To determine $\alpha(\omega)$ in this case, we have to solve Eqs. (4)–(7). Figure 1(a) shows two exemplary solutions calculated for $W/l_{sc}=2$ (dotted line) and $l_{sc}/W=4$ (stippled line). It is evident that as $l_{sc}/W\to 0$, the exactly calculated phase trajectory approaches the universal diffusion curve. On the other hand, in the limit $W/l_{sc}\to 0$ we recover the ballistic result (9).

It should be noted that phase trajectories, calculated from the BTE, always intersect the diffusion spiral at one point [Fig. 1(b)]. Inasmuch as the diffusion curve is universal, we can use the intersection point for a reference. The value of the phase $\gamma^x\equiv|\gamma_d|$ at the intersection varies with l_{sc}, diverging for $l_{sc}/W\to 0$ and tending to a finite limit $\gamma_b^x\approx 0.56$ [indicated in Fig. 1(b) by a vertical asymptote, dotted line] for $W/l_{sc}\to 0$. Measuring γ^x, one can determine the scattering length in the base of a given transistor with the help of Fig. 1(b).

It is worth emphasizing that the physical nature of the decay of $|\alpha_b|$ at high frequencies is quite different from that of $|\alpha_d|$. In the case of ballistic transport, the gain degradation results from the scatter in the velocities and the incident angles of the in-bound electrons. Members of a minority-carrier ensemble, injected into the base at a given time, that have different normal components of the velocity, arrive at the BC junction at different times. This has the effect of washing out any modulation of the injection current. A meaningful analogy can be drawn with the Landau damping of density waves in collisionless plasmas.[4]

Let us illustrate this damping process by evaluating α_b for initial distributions that are sharply peaked in either the incident angle or the energy:

$$f(0,k,u>0)\sim\delta(E_k-E_0),$$

$$\alpha_b=2E_3(i\gamma_0+W/l_{cp}),\tag{10}$$

$$f(0,k,u>0)\sim\delta(1-u)e^{-E_k/kT},$$

$$\alpha_b=e^{-W/l_{cp}}\int_0^\infty \epsilon e^{-\epsilon}e^{-i\gamma_b\sqrt{3/2\epsilon}}\,d\epsilon,\tag{11}$$

$$f(0,k,u>0)\sim\delta(1-u)\delta(E_k-E_0),$$

$$\alpha_b=e^{-i\gamma_0}e^{-W/l_{cp}},\tag{12}$$

where $\gamma_0=W\omega/v_0$ and $mv_0^2/2=E_0$. The behavior of α_b for these idealized distributions is illustrated in Fig. 2. We see that any initial scatter in either u or E_k makes the phase trajectory of α_b an inward-bound spiral. Only a truly collimated monochromatic beam, Eq. (12), does not decay in collisionless transport; its phase trajectory is a circle $|\alpha_b|=e^{-W/l_{cp}}$. As shown below, this situation is nearly approached in hot-electron ballistic transistors.[5]

In heterojunction bipolar transistors (HBT) the band structure can be engineered[6] in such a way that the minority carriers are injected into the base "over a cliff" of energy Φ_0. In this case, the appropriate in-bound distribution is of the form

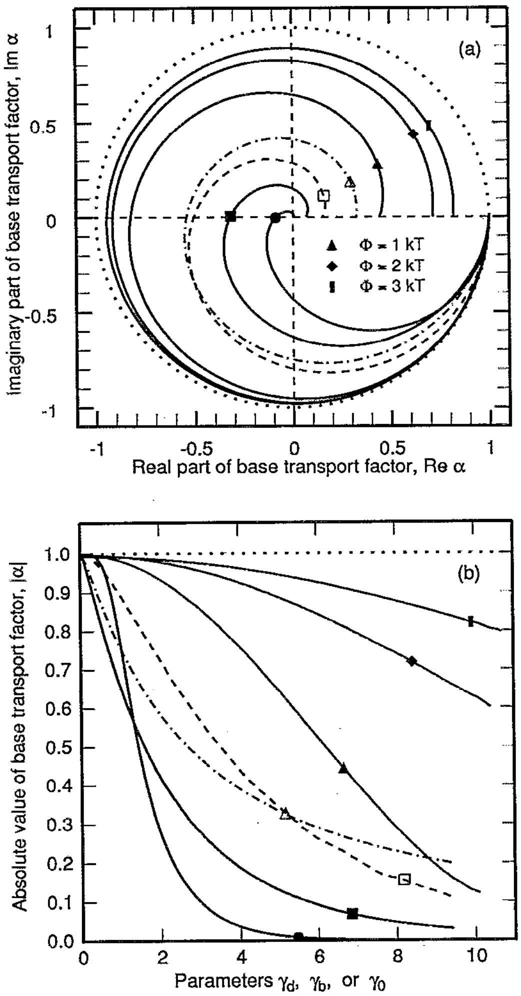

FIG. 2. Phase trajectories for HBTs. (a) The base transport factor $\alpha_b(\omega)$ evaluated from Eq. (14), is plotted for three different injection energies Φ. For convenience, on the same graph we present the α_b corresponding to $\Phi=0$ (homojunction) and the universal diffusion spiral α_d. These curves are labeled as in Fig. 1(a). The stippled, dashed, and dotted lines represent the α_b calculated for artificial initial distributions, corresponding, respectively, to the sharp energy [Eq. (10)], the sharp incident angle [Eq. (11)], and the collimated monochromatic beam [Eq. (12)]. (b) Magnitude $|\alpha|$ of the base transfer ratio plotted in each case against its "natural" phase parameter, which is γ_d for Eq. (8), γ_b for Eqs. (9), (11), and (14), and γ_0 for Eqs. (10) and (12). The symbols [consistent with Figs. 1(a) and 2(a)] mark the position where the phase value is $\arg(\alpha) = -2\pi$.

$$f(0,k,u>0)=e^{-\frac{E_k-\Phi_0}{kT}}, \quad \text{if } E_k>\Phi_0 \text{ and } u>\frac{k_0}{k}, \qquad (13a)$$

$$f(0,k,u>0)=0, \qquad \text{if } E_k<\Phi_0 \text{ or } u<\frac{k_0}{k}, \qquad (13b)$$

where $\hbar k_0 \equiv \sqrt{2m\Phi_0}$. Substituting Eqs. (13) in Eqs. (5)–(7) for the case $l_{sc}^{-1}=0$, we find

$$\alpha_b=2\int_{\phi_0}^{\infty} \epsilon \, d\epsilon e^{\phi_0-\epsilon} \int_1^{\sqrt{\epsilon/\phi_0}} t^{-3} \times \exp\left[\left(-i\sqrt{\frac{3}{2}}\frac{\gamma_b}{\sqrt{\epsilon}}-\frac{W}{l_{cp}}\right)t\right] dt, \qquad (14)$$

where $\phi_0 \equiv \Phi_0/kT$. Phase trajectories, calculated from Eq. (14) for $\phi_0=1$, 2, and 3, are shown in Fig. 2(a). For comparison, the same graph also shows the other universal trajectories found in this work. The dependence of the magnitude of $|\alpha|$ on the relevant phase parameter γ_d, γ_b, or γ_0 is presented in Fig. 2(b). In this plot, the symbols (triangles, squares, etc.) are placed on each curve at the position where the phase value is $\arg(\alpha) = -2\pi$.

An important point to note in Fig. 2 is the *strong temperature dependence* of the absolute transfer ratio $|\alpha(\omega)|$. That this should be the case is physically clear once it is recognized that the gain degradation in ballistic transistors is analogous to the "Landau damping," as discussed above. The main effect of temperature is to increase dispersion of the incident distribution.

We remark that it would not be too hard to evaluate $\alpha(\omega)$ for a HBT at a finite ratio W/l_{sc}, as we have done in Fig. 1 for homojunction transistors. However, such a calculation, based on the concept of a scattering length $l_{sc}(k)$, would not do justice to the hot-electron device problem, where the energy dependence of the electron interaction with optical phonons (and plasmons) requires a more refined treatment. Such a treatment will be reported in a subsequent publication. Moreover, in the case of an HBT it may become necessary to include the quantum-mechanical effect of above-barrier reflection of hot electrons returning to the abrupt emitter interface upon scattering in the base. It is clear that such processes are unimportant for homojunction transistors with or without scattering and that they need not be included in the above discussion of HBT in the ballistic limit.

Recently, there have been several attempts[7,8] to experimentally distinguish the diffusive and the ballistic transport by studying the base thickness dependence of static gain β. In a purely diffusive model the gain scales as $\beta \propto 1/W^2$, whereas in the ballistic limit $\beta \propto 1/W$. In our earlier work,[1] it was shown that homojunction transistors exhibit a similar behavior. The present analysis offers a possibility of distinguishing the dominant transport mechanism by studying high-frequency characteristics of a single transistor. Moreover, our method does not rely on any assumptions about the recombination processes that control the static gain.

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