Mid-Term Examination: ESE 558 Digital Image Processing
Date: 3/19/2002, Duration: 2 hours, Spring 2002
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This test is open-text books. No other reference materials are allowed. Note: G&W refers to the text book by Gonzalez & Woods. In answering the following questions write all steps.

1. (5 points) Ch. 2, G&W, Digital Image Fundamentals
   (a) (3 points) Explain how certain characteristics of the human eye provide it a large dynamic range for brightness sensing (e.g. the eye can see under very low illumination such as moon light and also very bright illumination such as sun light, but not simultaneously).
   (b) (2 points) A CCD camera chip of dimensions 5 mm X 5 mm, and having 1000 x 1000 elements is focused on a white board. The board is 500 mm from the camera lens and is perpendicular to the optical axis of the lens. The focal length of the camera is 10 mm. What is the maximum density of parallel lines drawn on the board (expressed as number of lines per mm) which the camera can resolve correctly? (Hint: see Excercise 2.5 on page 72 in the G&W text book).

2. (15 points) Ch. 3, G&W, Image Enhancement: Spatial domain
   (a) (7 points) The probability density functions $p_r(r)$ and $p_z(z)$ corresponding to the histograms of an input image R and output image Z are shown in Fig. 1 and Fig. 2.
   (i) (3 points) Find the transformation functions $s = T(r)$ and $v = G(z)$ that can be used to equalize the histograms of the two images.
   (ii) (3 points) Find the transformation $z = H(r)$ that can be used to transform the input image with histogram $p_r(r)$ so that the output image has the histogram $p_z(z)$ (Histogram specification).
   (iii) (1 point) If the brightness of a pixel in the input image is 0.4, what is the brightness of the same pixel in the output image (after histogram specification transformation)?
   (b) (3 points) For median filtering in $n \times n$ neighborhoods, propose a technique for updating the median as the center of the neighborhood is moved from a pixel to its right neighbor.
   (c) (2 points) Derive a spatial domain filter for estimating $\frac{\partial^2 f}{\partial x \partial y}$.
   (d) (3 points) (Laplacian of Gaussian: discrete domain)
   A digital image f(m,n) is first smoothed for noise reduction by convolution with an approximate 3x3 Gaussian filter g(m,n). Then the resulting image is convolved with a 3x3 Laplacian filter L(m,n). Given g(m,n) and L(m,n), derive the effective single filter h(m,n) that can be convolved with the image f(m,n) to obtain the same result as before (i.e. first convolving with g(m,n) and then with L(m,n)).

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            1     | 1   2   1 |                 | 0    1   0   |
g(m,n)= ---      | 2   4   2 |     L(m,n) =    | 1   -4   1   |
           16     | 1   2   1 |                 | 0    1   0   |
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3. (10 points), G&W Ch. 4 and R&K Ch. 2, Image Enhancement: Frequency domain

   (a) (5 points) State and prove the convolution theorem for the case of one-dimensional continuous Fourier transform. Do not assume any property of the Fourier transform. Prove any property as needed.

   (b) (3 points) f(m,n) is a discrete image of size MxN, and F(u,v) is its Discrete Fourier Transform (DFT). Starting from the definition of DFT, prove that $F(aM+u, bN+v) = F(u,v)$ where a and b are integer constants.

   (c) (2 points) Using the continuous Frequency domain transfer functions for the Gaussian and the Lapalcian in the text book, derive an expression for the transfer function of the *Laplacian of the Gaussian* (that is, first filtering with the Gaussian, and then the Laplacian). Draw a qualitative (subjective) plot of the resulting transfer function. Assume some reasonable value for the Gaussian parameter.