Mid-Term Test 2: ESE 558 Digital Image Processing

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This test is open text book. Only the text books are permitted. No other materials are permitted. Show all steps to demonstrate your understanding and to get full credit.

1. *Fourier domain enhancement*: (6=3+3 points)

   (a) (3 points) Let $f(m,n)$ be a digital image of size $M \times N$ and $F(u,v)$ be its Discrete Fourier Transform (DFT). Prove the following periodicity property of the DFT:

   $$F(u,v) = F(aM+u, bN+v) \tag{1}$$

   where a and b are integers.

   (b) (3 points) Prove the *convolution property* of the one-dimensional continuous Fourier transform. Show all steps. In particular, do not use other properties of the Fourier transform to skip steps.

2. *Sampling theorem:* (6=1+4+1 points)
   An image is specified by $f(\mathbf{r}) = \cos 2\pi(\mathbf{w_0} \cdot \mathbf{r})$ where $\mathbf{r} = (x,y)$ and $\mathbf{w_0} = (0.3, 0.8)$. The image is sampled on a lattice $\mathbf{r_{mn}} = m\mathbf{r_1} + n\mathbf{r_2}$ where $\mathbf{r_1} = (1,0)$, and $\mathbf{r_2} = (0,1)$ for $m,n = 0, \pm1, \pm2, \pm3, \cdots$..

   (a) Find the reciprocal lattice $\mathbf{w_{mn}} = m\mathbf{w_1} + n\mathbf{w_2}$ (i.e. find $\mathbf{w_1}$ and $\mathbf{w_2}$).

   (b) If $G(\mathbf{w}) = \text{rect}(\mathbf{w})$ is the Fourier transform of the interpolation filter $g(\mathbf{r})$ used in reconstructing the image from its samples, find an explicit expression for the reconstructed image $f'(\mathbf{r})$.

   (c) Suppose that the interpolation filter is changed to $G(\mathbf{w}) = \text{Circ}(\mathbf{w})$ whcih has as value of 1 inside a circle centered at the origin and of radius 0.5, and zero outside. In this case, will the reconstructed image change? Explain your answer.

3. *Optimal quantization: (3 points)*
   The probability density function of an image signal z is

   $$\begin{aligned} p(z) &= \frac{2}{3}\left(1-|z|^3\right) \text{ for } |z| \leq 1, \text{ and} && (2)\\ &= 0 \text{ for } |z| > 1. && (3)\end{aligned}$$

   We need to optimally quantize the signal to two distinct levels q_1, q_2 with a decision level z_2. Guess a solution for z_2 using common sense and then solve for q_1, q_2.

4. *Image Restoration:* (4+3+3 points)
   The point spread function $h(x,y)$ of a misfocused video camera with a square aperture is

   $$h(x,y) = \frac{1}{4}\text{rect}(x/2, y/2) \tag{4}$$

where

$$\begin{aligned} \mathrm{rect}(x/2, y/2) &= 1 \text{ for } -1 \leq x, y \leq 1 && (5) \\ &= 0 \text{ otherwise.} && (6) \end{aligned}$$

(a) (4 points) Give an expression for restoring the blurred image $g(x, y)$ to obtain the focused image $f(x, y)$ using the Spatial-domain convolution/deconvolution transform. Note: Assume that Equation (6) on page 5 in the journal paper is valid. You should derive $h_{2,0}$ and $h_{0,2}$.

(b) (3 points) In this problem, suppose that the Signal to Noise ratio is given by

$$SNR = \frac{|F(u, v)|^2}{|N(u, v)|^2} = A\, e^{-B(u^2+v^2)} \tag{7}$$

where A, B are scalar constants. Give an explicit expression for restoring the blurred image using the Weiner filter. Your filter should express $H(u, v)$ explicitly in the Weiner filter.

(c) (3 points) If the same video camera is misfocused as above, and further, in addition to misfocus, motion blur is caused by the motion of the object. Let the motion be such that the image of the object moves uniformly by 1 mm along the x-axis during the exposure period of 0.1 second. Give an expression for the image restoration filter based on Weiner filter. Your single filter should simultaneously undo/deblur the effects of both misfocus and motion blur. Make your answer as specific and explicit as possible. You need not derive anything in this question. You can use any results given the text books.