PH.D. QUALIFYING EXAMINATION STUDENT'S NAME ____________________

COMMUNICATIONS AND SIGNAL PROCESSING APRIL 2003

QUESTION #7 (10 points) Image reconstruction from parallel projections:

1. (5 points) Let $P_\theta(t)$ be the parallel projection (Radon Transform) of a continuous image function $f(x, y)$ at an angle θ degrees. Given

$$f(x,y) = \begin{cases} x^2 + y^2 & \text{if } -1 \leq x \leq 1 \text{ and } -1 \leq y \leq 1, \text{ and} \\ 0 & \text{otherwise,} \end{cases}$$

find the Radon Transform $P_0(t)$, i.e. find the projection onto the x-axis as $\theta = 0$ degrees.

2. (5 points) For the Filtered-Backprojection algorithm for image reconstruction from parallel projections, list the input, output, and the computational steps necessary to transform the input data to output data.

USE THIS SPACE FOR YOUR ANSWER

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QUESTION #8

There are three independent questions below. The third is weighted most heavily.

1. A pinhole camera imaging system views a 3D scene and records an image. Considering the scene as input and and the image as output, is this a linear shift-invariant imaging system? Explain briefly.

2. The 2D (continuous) Fourier transform is a widely used analytic tool in image processing. It maps a space domain function $f(x, y)$ to its frequency-domain transform $F(\xi, \eta)$. Is this transfrom linear? Is it shift invariant? Explain.

3. A continuous 2D imaging system delivers an image $f(x, y)$ to a sensor. The quantity $f(x, y)$ is bandlimited in all directions to a max spatial frequency ρ_c. If $f(x, y)$ were sampled at the Nyquist frequency, it could be recovered using the Sampling Theorem. However, the sensor first blurs (i.e. convolves, denoted by $*$) the image by a Gaussian $p(x, y)$ of variance σ^2. The resulting blurred quantity $g(x, y) = f(x, y) * p(x, y)$ is *then* sampled by the detector elements at the Nyqusit frequency to form a digital image. Show how one could recover $f(x, y)$ from these digital samples. (Note that this problem is unrealistic in that (a) an infinite number of samples are needed and (b) real detectors do not form ideal delta-function samples of the sort discussed in textbooks.)

It is important to be especially clear about your notation and definitions for all parts of the problem.

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