QUALIFYING EXAMINATION STUDENT'S NAME ____________________

SIGNAL PROCESSING APRIL 2000

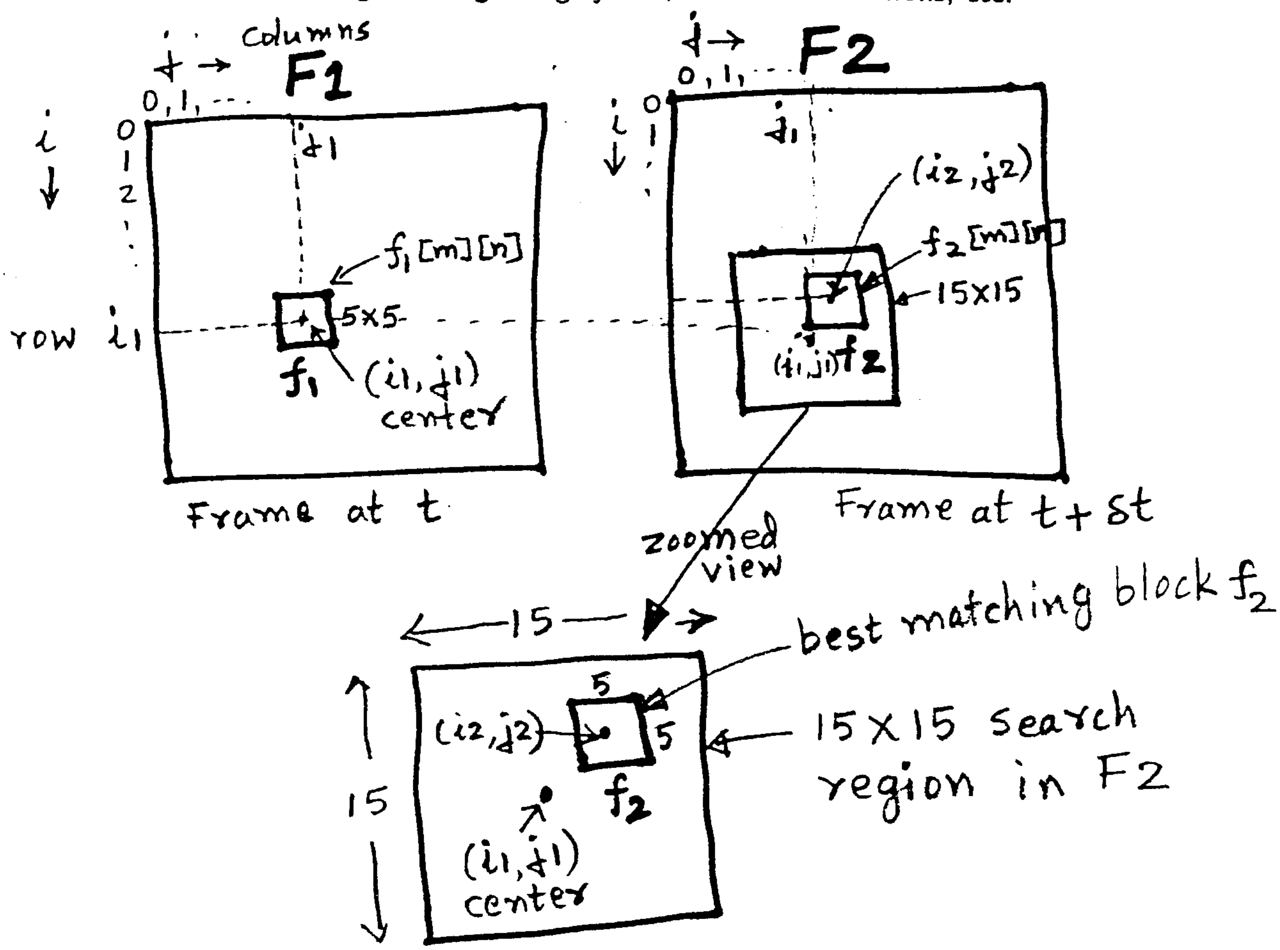
QUESTION # 5

(10 points)

You are given two consecutive or successive digital image frames F1[i][j] and F2[i][j] in a digital video stream. The image frames F1 and F2 differ by a small amount due to moving objects in the video. Consider an 5×5 image block f1[m][n] in F1 (see Figure) centered at (i1,j1). The motion of this block is estimated by searching for a best matching block f2[m][n] in F2 in a 15×15 size image region centered at (i1,j1) in F2. The search is made by computing the difference measure

$$D = \sum_{m=1}^{5} \sum_{n=1}^{5} (f1[m][n] - f2[m][n])^2 \tag{1}$$

for every possible 5×5 candidate block f2[m][n] in the 15×15 image region in F2. The best matching block f2[m][n] is that with the minimum difference measure D. Suppose that the center of the best matching block f2[m][n] in F2[i][j] is at (i2,j2). Write a computational algorithm to find (i2,j2) given (i1,j1). Assume that (i1,j1) is an interior point more than 8 units from the border of F1 on all sides so that your algorithm becomes simple (there is no need to deal with border cases). You can write the algorithm in any high-level programming language (C/Fortran/Pascal/etc.) or english psuedo-code. Just write the core of the algorithm ignoring syntax, variable declarations, etc.



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QUESTION # 6

Please use additional sheets as needed. Show all work. Define all terms and symbols.

(Note: 1-D means "one-dimensional" and 2-D "two-dimensional")

a) [3 points] The Laplacian operator ∇^2 is often used for 2-D (continuous) edge enhancement . If $s(x,y)$ is the enhanced version of input image $f(x,y)$, then the space domain operation $s(x,y) = \nabla^2 f(x,y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$ may be viewed in the Fourier domain as multiplication of the input image spectrum by a transfer function. What is the transfer function?

b) [2 points] In the discrete domain, the Laplacian operator can take the form of a 2-D convolution of a discrete 2-D input f_{ij} with a 3x3 template. Below are several choices for the template. Which one best approximates the Laplacian? Which one would best approximate the operation of enhancing horizontal edges? Clearly indicate your choice for each of the two questions above. Discussion is optional.

$$\underset{(1)}{\begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}} \quad \underset{(2)}{\begin{bmatrix} 1 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 1 \end{bmatrix}} \quad \underset{(3)}{\begin{bmatrix} -1 & 0 & -1 \\ -1 & 0 & -1 \\ -1 & 0 & -1 \end{bmatrix}} \quad \underset{(4)}{\begin{bmatrix} -1 & -1 & -1 \\ 2 & 2 & 2 \\ -1 & -1 & -1 \end{bmatrix}} \quad \underset{(5)}{\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}}$$

c) [2 points] For a 1-D discrete problem, the N-pixel input image may be represented as an N-dimensional vector $\mathbf{f}$. The discrete 3×3 Laplacian of part (b) now collapses to the 3-element vector $\mathbf{h} = (-1, 2, -1)$. Thus the edge enhanced $\mathbf{f}$ is given by $\mathbf{s} = \mathbf{f} * \mathbf{h}$, where $*$ denotes discrete 1-D convolution. Actually, one can rewrite this convolution as a matrix-vector product $\mathbf{s} = \mathcal{H}\mathbf{f}$. For N=5, write down the elements of matrix $\mathcal{H}$. Note that the matrix can be square or rectangular depending on your assumptions. State all assumptions.

d) [3 points] Consider the 1-D edge-enhancement problem in part (c). In practice, the edge-enhancement operator $\mathcal{H}$ would be applied to a *noisy* input. Let's say that the input $\mathbf{f}$ is corrupted by the addition of i.i.d. (independent identically distributed) Gaussian noise with mean zero and variance σ^2. The output image $\mathbf{s}$, now a random vector, results from multiplying this noisy input by $\mathcal{H}$. What is the pdf (probability density function) of $\mathbf{s}$?

USE THIS SPACE FOR YOUR ANSWER