On the Depth Information in the Point Spread Function of a Defocused Optical System

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Abstract

The Point Spread Function (PSF) of an image forming optical system is determined by the parameters of the optical system and the distance or depth of the object being imaged. In this paper we consider the depth information in the PSF of the optical system determined by paraxial wave optics under quasi-monochromatic incoherent illumination. The *spread parameter* of a PSF is defined as the *standard deviation* of the distribution of the PSF. If σ_w is the spread parameter of the PSF of a defocused optical system corresponding to an object at distance u, then it is shown that $\sqrt{\sigma_w^2 - \sigma_0^2} \approx mu^{-1} + c$ where σ_0^2, m, and c are constants determined by the parameters of the optical system. This relation is useful in determining the distance u of the object using its defocused image. Usually, $\sigma_w >> \sigma_0$, and therefore the linear relation $\sigma_w \approx m\,u^{-1} + c$ is obtained between σ_w and $1/u$. In this paper, we also compare the PSFs determined by paraxial wave optics and paraxial geometric optics.

1 Introduction

The defocus information in the image of an object formed by a camera system can be used to determine the distance or depth of the object from the camera system [9, 3, 12, 13, 14, 15]. Here we present some results on the relation between a measure of defocus and the distance of the object.

The Point Spread Function (PSF) of an image forming optical system is the image of a point light source object. The PSF depends both on the parameters of the optical system and the distance or depth of the point light source object from the optical system. Here we consider the depth information contained in the PSF corresponding to a quasi-monochromatic and incoherent point light source object. The angle θ between the direction of location of the point object and the optical axis is assumed small so that the paraxial approximation $\sin\theta \approx \theta$ is acceptable.

We characterize a PSF in terms of a *spread parameter* defined as the standard deviation of the distribution of the PSF. The spread parameter of the PSF of an optical system is a minimum when the optical system is perfectly focused. For defocused optical systems, the spread parameter increases with increasing defocus.

Let σ_w and σ_o be the spread parameters of PSFs corresponding to defocused and focused cases respectively where the PSFs are determined by paraxial wave optics. Further, let σ_g be the spread parameter of the PSF for defocused case when the PSF is determined by paraxial geometric optics. In this paper we show that $\sigma_w^2 = \sigma_g^2 + \sigma_0^2$. Consequently, we show that $\sqrt{\sigma_w^2 - \sigma_o^2} \approx m\,u^{-1} + c$ where u is the distance from the optical system of the object being imaged, and, m and c are constants determined by the parameters of the optical system. For a given optical system, σ_0 is a constant and is usually small (approximately 1.0) in comaprison with σ_w and therefore we get $\sigma_w \approx m\,u^{-1} + c$. This relation implies that the spread parameter is a linear function of the reciprocal of distance u. This relation had been derived earlier [15, 13] based on the principles of paraxial geometric optics; now it has been derived from the principles of paraxial wave optics.

The results above also confirm an earlier hypothesis based on experimental observations [15, 13] that the spread parameter of a PSF is approximately proportional to the diameter of the blur circle predicted by paraxial geometric optics.

We also compare here the PSF determined by paraxial geometric optics with that determined by paraxial wave optics for different amounts of defocus through computer simulation. Such a comparative study has been carried out earler by Lee [7]. Our comparative study shows that, as the amount of defocus increases, the PSF determined by paraxial wave optics approaches that determined by paraxial geometric optics. This result is in agreement with that obtained by Lee [7].

The derivations in this paper are based on a formula for the Optical Transfer Function (OTF) of a defocused optical system derived by Hopkins in his classic paper [5]. A very useful simplification of Hopkins' formula was given by Levi and Austing [8]. This simplified formula is the starting point of one of our main derivations. However, Hopkins' original derivations make certain approximations (which are well accepted in optics literature) and therefore, the formula for OTF used by us is not exact but only "almost exact". Because of this lack of exactness, the OTF formula implies that the value of the spread parameter of a perfectly focused optical system is infinity. This creates a problem in our derivation. We overcome this difficulty by showing through computer simulation that the spread parameter of the PSF of a perfectly focused optical system computed over a large but finite region on the image detector plane has a finite value (approximately 1.0). Physical arguments also can be made to support that the spread parameter has to be finite in value.

2 Defocusing Model

The defocusing model used here follows from the models used by Hopkins [5], Levi and Austing [8], and Stokseth [11] in particular.

The properties of an image forming optical system can be described by specifying the electro-magnetic field distribution at the *entrance pupil* and the *exit pupil* of the optical system ([2], Section 6-1; [1]). In this paper we will consider only circularly symmetric optical systems, and therefore circular entrance and exit pupils.

Figure 1 shows a defocused imaging system. Here, O is a point object and WO is a diverging spherical wavefront emanating from O and incident on the entrance pupil ENP. The incident spherical wavefront WO is converted by the optical system into a new spherical wavefront WI emerging from the exit pupil EXP. The emerging wavefront WI converges toward an ideal image point I in the image space. If an image detector such as a photographic film is placed at I, then a "focused image" of the object point O is recorded by the film. If, however, the image detector (ID in Fig. 1) is placed at a different point such as P, then a "defocused image" or "blurred image" is recorded by the film.

Let WP be a hypothetical spherical wavefront at the exit pupil which converges to point P. We refer to WP as the *reference wavefront.* The path difference between the actual wave front WI and the reference wavefront WP as a function of angle θ (see Fig. 1) is called the wavefront aberration function W. The function W has its maximum value W_{max} (equal to length AB in Fig. 1) at the edge of the exit pupil. This maximum value is a measure of the *defect of focus.* Rayleigh's tolerance on defocusing is $\lambda/4$, and hence is taken as the unit of defocus. The value of focus defect (or the amount of defocus) is denoted by Δ and is defined by

$$\Delta = \frac{W_{max}}{\lambda/4}. \tag{1}$$

In Figure 1, let θ be the half-angle of the cone subtended by the exit pupil at the focused image point I. We will restrict our analysis to only defocus aberration. Other aberrations will be assumed to be negligible. Therefore θ will be assumed to be small so that the approximation $\sin\theta \approx \theta$ is acceptable. Let R be the radius of the exit pupil, v the radius of wavefront WI, s the radius of wavefront WP, δ the distance between the focused image point I and the image detector position P.

All quantities will be expressed in reduced units so that the results can be stated independent of wavelength and the f-number of the optical system. First, geometrical lengths are normalized by the radius R of the exit pupil. Next, the distances measured on the image detector plane are normalized by half the distance of the image detector from the exit pupil ($s/2$ in Fig. 1) and the wavelength λ. This has the effect of normalizing the image magnification and then normalizing with respect to λ. As a consequence of these three normalization steps, if r' is the geometrical length measured on the image detector and r is the reduced length, then

$$r = \frac{r'}{(s/2R)\,\lambda} = \frac{r'}{F\,\lambda} \tag{2}$$

where F is the f-number of the imaging system defined by

$$F = \frac{s}{2R}\ . \tag{3}$$

Also, if ρ' is the actual spatial frequency of an image pattern measured on the image detector plane, then the reduced spatial frequency ρ is given by

$$\rho = \lambda\,\rho'\,F\ . \tag{4}$$

3 Defocus Δ

For triangle IPB in Fig. 1, noting that $\cos(\pi - \theta) = -\cos\theta$ and using the cosine law of triangles, we have

$$BP^2 = v^2 + \delta^2 + 2v\delta\cos\theta \ . \tag{5}$$

For paraxial optics, θ is small, and therefore

$$\theta \ \approx \ \frac{R}{v} \ . \tag{6}$$

In addtition, by ignoring fourth and higher order terms in the series expansion of $\cos\theta$, we obtian

$$\cos\theta \ \approx \ 1 - \frac{R^2}{v^2} \ . \tag{7}$$

From equations (5) and (7) we obtain

$$BP \ \approx \ (v+\delta)\left[1 - \frac{\delta R^2}{v(v+\delta)^2}\right]^{1/2} \ . \tag{8}$$

Assuming $|\delta| \ll 1$ in equation (8) we get

$$BP \ \approx \ (v+\delta)\left[1 - \frac{\delta R^2}{2v(v+\delta)^2}\right] \ . \tag{9}$$

In Fig. 1,

$$W_{max} = AB = AP - BP = v + \delta - BP \ . \tag{10}$$

From equations (10), (9), (1), and noting that $s = v + \delta$, we obtain

$$\Delta \ \approx \ \frac{2R^2}{\lambda}\,\frac{\delta}{v(v+\delta)} \ = \ \frac{2R^2}{\lambda}\left(\frac{s-v}{v\ s}\right) \ = \ \frac{2R^2}{\lambda}\left(\frac{1}{v} - \frac{1}{s}\right) \ . \tag{11}$$

According to the well-known lens makers formula, we have

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \ . \tag{12}$$

From equations (11) and (12) we get

$$\Delta \ \approx \ \frac{2R^2}{\lambda}\left(\frac{1}{f} - \frac{1}{u} - \frac{1}{s}\right) \ . \tag{13}$$

Relation (13) gives an expression for the focus defect Δ in terms of the parameters s, f, R of the optical system, and the distance u of the object.

4 Relation between Δ and σ_g

According to paraxial geometric optics, the defocused image of a point is a circular disc of constant brightness called *blur circle*. In Fig. 1, the *blur circle* corresponding to object point O is centered at P on the image detector plane ID and has a radius $r'_o = PQ$. An expression for r'_o can be obtained by noting that, for small values of θ, triangles CO_2I and QPI are similar. We therefore have

$$\frac{r'_o}{\delta} = \frac{R}{v} \; . \tag{14}$$

If r'_o is the radius of the blur circle expressed in reduced units, then from relation (2) we obtain

$$r_o = \frac{r'_o}{\lambda(s/2R)} \; . \tag{15}$$

From equations (11),(14), and (15) we obtain

$$r_o = \Delta \; . \tag{16}$$

Therefore, the defocus parameter Δ is equal to the radius of the corresponding blur circle expressed in reduced units. The blur circle is indeed the PSF determined by the paraxial geometric optics. If $h_g(r)$ denotes this PSF, then we have

$$\begin{aligned} h_g(r) &= \frac{1}{\pi\Delta^2} \qquad \text{for} \quad 0 \leq r \leq \Delta \\ &= 0 \qquad\quad \text{for} \quad r > \Delta \; . \end{aligned} \tag{17}$$

For any circularly symmetric PSF $h(r)$, the spread σ of the PSF is defined as

$$\sigma^2 = 2\pi \int_0^\infty r^3 \, h(r) \, dr \; . \tag{18}$$

If σ_g is the spread parameter of $h_g(r)$, then from equations (17) and (18) we obtain

$$\sigma_g = \frac{\Delta}{\sqrt{2}} \; . \tag{19}$$

Above we have a relation between σ_g and Δ. From equations (13) and (19) we have

$$\sigma_g \approx m \, u^{-1} + c \tag{20}$$

where

$$m = -\frac{\sqrt{2}\,R^2}{\lambda} \quad \text{and} \quad c = \frac{\sqrt{2}\,R^2}{\lambda}\left(\frac{1}{f} - \frac{1}{s}\right) \; . \tag{21}$$

Equations (20) and (21) imply that σ_g is linearly related to the reciprocal of distance for a given set of parameters s, f, R of the optical system and wavelength λ.

5 Relation between σ_w and σ_g

Let $h_w(r)$ and $h_o(r)$ be the PSFs of the optical system determined by paraxial wave optics, where $h_w(r)$ corresponds to the defocused case (i.e. $\Delta > 0$), and $h_o(r)$ corresponds to the perfectly focused case (i.e. $\Delta = 0$). Let σ_w and σ_o be the spread parameters of $h_w(r)$ and $h_o(r)$ respectively. We shall now show that $\sigma_w^2 = \sigma_g^2 + \sigma_o^2$.

let $h_w(x, y)$ be the PSF and $H(\omega, \nu)$ be the corresponding optical transfer function, i.e. $H(\omega, \nu)$ is the Fourier transform of $h_w(x, y)$ given by

$$H(\omega, \nu) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} h(x, y)\, e^{-j2\pi(\omega x + \nu y)}\, dx\, dy\ . \tag{22}$$

Assuming $h_w(x, y)$ to be circularly symmetric, the spread parameter σ_w is given by

$$\sigma_w^2 = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (x^2 + y^2) h(x, y)\, dx\, dy\ . \tag{23}$$

From equations (22) and (23) it can be shown that

$$\left[\nabla^2 H(\omega, \nu)\right]_{\omega=\nu=0} = -4\pi^2 \sigma_w^2 \tag{24}$$

where ∇^2 is the Laplacian operator defined by

$$\nabla^2 = \frac{\partial^2}{\partial \omega^2} + \frac{\partial^2}{\partial \nu^2}\ . \tag{25}$$

Because the PSF $h_w(x, y)$ is circularly symmetric, the OTF $H(\omega, \nu)$ is also circularly symmetric. Therefore we will denote the PSF by $h_w(r)$ where $r = \sqrt{x^2 + y^2}$ and OTF by $H(\rho)$ where $\rho = \sqrt{\omega^2 + \nu^2}$. According to Levi and Austing [8] [1], the OTF corresponding to a focus defect Δ is given by

$$H(\rho, \Delta) = \frac{4}{\pi} \int_\rho^1 \sqrt{1 - t^2}\ \cos[2\pi\Delta\rho(t - \rho)]\, dt\ . \tag{26}$$

[1] There is a typographic error in the Levi and Austing [8] paper; the factor $\frac{4}{\pi a}$ should be $\frac{4}{\pi}$.

In polar coordinates, equations (24) and (25) can be written as

$$\left[\nabla^2 H(\rho,\Delta)\right]_{\rho=0} = -4\pi^2\sigma_w^2 \ , \tag{27}$$

and

$$\nabla^2 = \frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} \ . \tag{28}$$

To find the Laplacian of $H(\rho,\Delta)$, we need to take derivatives of the right hand side (RHS) of equation (26) with respect to ρ. Taking derivatives is not striaghtforward since ρ appears both in the integration limit and in the function to be integrated . However, the following trick [2] can be used. Consider two functions $\rho_1(\rho)$ and $\rho_2(\rho)$ given by

$$\rho_1(\rho) = \rho \quad \text{and} \quad \rho_2(\rho) = \rho \ . \tag{29}$$

Now equation (26) can be written as

$$H(\rho_1,\rho_2;\Delta) = \frac{4}{\pi}\int_{\rho_1}^{1} \sqrt{1-t^2}\cos[2\pi\Delta\rho_2(t-\rho_2)]\,dt \ . \tag{30}$$

Now we can use chain rule to find the derivatives of H with respect to ρ. For example,

$$\begin{aligned}\frac{\partial H}{\partial\rho} &= \frac{\partial H}{\partial\rho_1}\frac{d\rho_1}{d\rho} + \frac{\partial H}{\partial\rho_2}\frac{d\rho_2}{d\rho} \\ &= \frac{\partial H}{\partial\rho_1} + \frac{\partial H}{\partial\rho_2} \ . \end{aligned} \tag{31}$$

Similarliy we can show that

$$\frac{\partial^2 H}{\partial\rho^2} = \frac{\partial^2 H}{\partial\rho_1^2} + \frac{\partial^2 H}{\partial\rho_1\partial\rho_2} + \frac{\partial^2 H}{\partial\rho_2\partial\rho_1} + \frac{\partial^2 H}{\partial\rho_2^2} \ . \tag{32}$$

From equations (30), (31), and (32) we can derive

$$\frac{\partial H}{\partial\rho_1} = -\frac{4}{\pi}\sqrt{1-\rho_1^2} \tag{33}$$

$$\frac{\partial^2 H}{\partial\rho_1\partial\rho_2} = 0 \tag{34}$$

[2] This approach was suggested by Mr. Peter Meigom at S.U.N.Y Stony Brook.

$$
\begin{aligned}
\frac{\partial^2 H}{\partial \rho_1^2} &= \frac{4}{\pi}\frac{\rho_1}{\sqrt{1-\rho_1^2}} && (35)\\
\frac{\partial H}{\partial \rho_2} &= -8\Delta\int_{\rho_1}^{1}\sqrt{1-t^2}(t-2\rho_2)\sin[2\pi\Delta\rho_2(t-\rho_2)]dt && (36)\\
\frac{\partial^2 H}{\partial \rho_2\partial \rho_1} &= 0 && (37)\\
& && (38)
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial^2 H}{\partial \rho_2^2} = & -8\Delta\int_{\rho_1}^{1}\sqrt{1-t^2}\Big[[(t-2\rho_2)^2\cos[2\pi\Delta\rho_2(t-\rho_2)](2\pi\Delta) \\
& -2\sin[2\pi\Delta\rho_2(t-\rho_2)]\Big]\,dt \ . \qquad (39)
\end{aligned}
$$

Using the above expressions (28-39), and the result

$$\int_0^1 \sqrt{1-t^2}\ t^2 dt = \frac{\pi}{16}\ , \tag{40}$$

we obtain

$$\Big[\nabla^2 H(\rho)\Big]_{\rho=0} = -2\pi^2\Delta^2\ -4\pi^2\sigma_o^2\ , \tag{41}$$

where

$$\sigma_o^2 = \frac{1}{\pi^3}\lim_{\rho\to 0}\frac{\sqrt{1-\rho^2}}{\rho}\ . \tag{42}$$

From equations (27), (41), and (19) we obtain

$$\sigma_w^2 = \sigma_g^2 + \sigma_o^2\ . \tag{43}$$

From equations (27) and (41), we see that when $\Delta = 0$, $\sigma_w = \sigma_o$. Therefore, σ_o is the spread parameter of the PSF when the optical system is focused, i.e. when the focus defect Δ is zero. However, according to equation (42), σ_o is infinity.

On physical grounds, we believe that the spread parameter σ_o for a focused PSF (i.e. $\Delta = 0$) must be finite. The reason for σ_o becoming infinity in equation (42) is because the original derivation of Hopkins [5] on which equation (26) is based involves certain approximations, and therfore equation (26) is not exact.

The intensity distribution produced by a point light source on the image detector must decrease at least as the inverse of the squared radial distance (

inverse square law for light energy) for sufficiently large distances. Further, since a point light source produces only finite energy, equation (23) implies that σ_o must be finite. Computer simulations show that when σ_o is computed by carrying out the integration on the RHS of equation (23) over an area of the image plane accounting for 95% of the energy in the PSF, the value of σ_o is approximately 1.0. More details of the computer simulations are given in Section 6.

From equations (43) and (20) we have

$$\sqrt{\sigma_w^2 - \sigma_o^2} \;\approx\; m\,u^{-1} + c\;. \tag{44}$$

Above we have an equation relating the spread parameter σ_w to the distance u of the object. Note that σ_o^2, m, and c are constants for a given optical system, and therefore, one can estimate the distance u from a knowledge of σ_w.

In practice it is found that $\sigma_o \approx 1.0$ and $\sigma_w >> \sigma_o$. In this case we have

$$\sigma_w \;\approx\; m\,u^{-1} + c \tag{45}$$

which gives a direct linear relation between σ_w and u^{-1}. Equation (45) implies that

$$\sigma_w \;\approx\; \sigma_g\;. \tag{46}$$

Relations (46), (16), and (19) together confirm the earlier hypothesis of Subbarao and Natarajan [15] based on experimental observations that the spread parameter of a PSF is approximately proportional to the diameter of the corresponding blur circle.

It is interesting to cross check equation (26) given by Levi and Austing [8] with that in Goodman [2] when $\Delta = 0$ (equation 6.31 in [2]). Substituting $\Delta = 0$ in the equation (26), we get

$$H(\rho, 0) = \frac{4}{\pi}\int_\rho^1 \sqrt{1-t^2}\,dt\;. \tag{47}$$

The RHS above can be integrated to obtain

$$H(\rho, 0) = \frac{2}{\pi}\left[\frac{\pi}{2} - \sin^{-1}\rho - \rho\sqrt{1-\rho^2}\right]\;. \tag{48}$$

Noting that $\frac{\pi}{2} - \sin^{-1}\rho = \cos^{-1}\rho$, we obtain

$$H(\rho, 0) = \frac{2}{\pi}\left[\cos^{-1}\rho - \rho\sqrt{1-\rho^2}\right]\;. \tag{49}$$

The above equation is exactly the same as that in [2].

6 Comparison of PSFs for geometric optics and wave optics

The PSF for paraxial geometric optics is given by equation (17). For paraxial wave optics, the PSF is given by inverse Fourier-Bessel transform as below:

$$h_w(r,\Delta) = 2\pi \int_0^\infty H(\rho,\Delta) J_0(2\pi\rho r)\rho \, d\rho \ , \tag{50}$$

where J_0 is the zeroth order Bessel function of the first kind, and r is the reduced radial distance on the image detector plane.

The OTF $H(\rho,\Delta)$ was computed for different amounts of defocus defect ($\Delta = 0, 1, 2, 5, 10, 30, 50, 70, 100, 200, 500, 1000$) using equation (26). A numerical integration method (trapezoidal rule) was used to integrate equation (26). A plot of the OTFs thus computed is shown in Fig. 2. Such plots of OTFs for defocusing can be found in many papers and books ([1] page 486-487 ; [5] ; [11]). We have included these plots for completeness.

The OTFs computed above were used to compute the corresponding PSFs using eqaution (50). Again a numerical integration method was employed [3]. A plot of the computed PSFs are showen in Fig. 3. We see that, for small values of defocus Δ, the PSFs exhibit dark and bright rings predominantly. As defocus Δ increases, the width of the rings become narrower, and generally the contrast between dark and bright rings reduces. For large values of defocus Δ, the rings become very narrow with reduced contrast (thus becoming invisible) and the PSFs become very similar to the PSFs predicted by geometric optics. In each plot, both diffraction PSF $h_w(r)$ and geometric PSF $h_g(r)$ are plotted so that the two can be compared. We have found such plots of PSFs only in the work of Lee [7] and nowhere else. These plots are useful in gaining insight into the nature of the PSFs.

For the PSFs ($h_w(r)$s) computed above, the corresponding spread parameters (σ_ws) were computed using equation (18). Here again, a numerical integration method was used. The upper limit of the integration in equation (18) was taken to be a finite value L such that the "volume" (or energy) under the PSF $h_w(r)$ within a region of radius L centered at the origin was 95% or more of the total volume (or energy). If $V(L)$ denotes this volume, then

$$V(L) = 2\pi \int_0^L r \ h(r) dr \ . \tag{51}$$

[3] The upper limit of integration in equation (50) can be set to 1.0 because $H(\rho,\delta) = 0$ for $\rho > 1.0$.

Note that, for all PSFs,

$$\lim_{L \to \infty} V(L) = 1.0 \ . \tag{52}$$

Therefore, L was such that $V(L) \geq 0.95$.

Table 1 shows the computed values of σ_w for various defocus amounts Δ. In addition, the table also lists the corresponding values of $V(L)$, σ_g (or $\Delta/\sqrt{2}$), and $\sqrt{\sigma_w^2 - \sigma_o^2}$.

First we point out that σ_o has a finite value which is in fact relatively small (approximately 1.0). All computed values of σ_w listed are less than the actual values of σ_w because $V(L) < 1.0$. Acoording to our theoretical results in the earlier sections, $\sqrt{\sigma_w^2 - \sigma_o^2}$ should be approximately equal σ_g. We see that this is indeed true within computational approximations (i.e. errors caused by $V(L) < 1.0$ and by numerical integration, and limited precision of computations). Actual experimental verification of the relation $\sqrt{\sigma_w^2 - \sigma_o^2} \approx m\, u^{-1} + c$ can be found in [14]. A plot of $\sqrt{\sigma_w^2 - \sigma_o^2}$ vs Δ is shown in Fig. 4 at three different scales. As predicted by theory, we see that the plots are almost straight lines with slopes close to $1/\sqrt{2}$ at all scales.

7 PSF for broad-band illumination

Until now we have considered the PSF due to quasi-monochromatic illumination. For broad-band illumination, if $I(\lambda)$ is the illumination spectrum, then the overall PSF is given by

$$h(r) = \int_0^\infty I(\lambda)\, h_w(r, \lambda)\, d\lambda \ . \tag{53}$$

As a simple example, consider the PSF of human eyes under white light illumination. Let $\Delta_o = 10$ for $\lambda_o = 450$ nm. A typical visibility curve for a normal eye is shown in Fig. 5 (after [10]). A discrete approximation to the overall PSF can be computed as follows. Let W_i be the scaled value of the visibility curve at $\lambda_i = 450 + i * 20$ nm for $i = 0, 1, 2, \cdots, 10$. From equation (13) we compute the defocus Δ_i for different λ_i as

$$\Delta_i = \Delta_o \, \frac{\lambda_0}{\lambda_i} \ . \tag{54}$$

For each Δ_i, we compute $h_i(r)$ using equation (50). The weights W_i are normalized such that they add to 1.0, i.e.

$$\sum_{i=0}^{10} W_i = 1.0 \ . \tag{55}$$

Now the overall PSF can be estimated as

$$h(r) = \sum_{i=0}^{10} W_i \; h_i(r) \; . \tag{56}$$

This overall PSF can be compared with the PSF predicted by geometric optics where the average defocus amount $\overline{\Delta}$ is estimated by

$$\overline{\Delta} = \sum_{i=0}^{10} W_i \; \Delta_i \; . \tag{57}$$

The results of these comptutations are shown in Fig. 6. Copmaring this overall PSF for white light illumination to the corresponding quasi-monochromatic PSF in Fig. 3 ($\Delta = 10$) we see that the two are different.

Another result of computing the overall PSF for white light illumination is shown in Fig. 7. In this case λ_is are the same as in the previous case, but $\Delta_o = 100$, and $W_i = \frac{1}{11}$ for all i (i.e. uniform visibility). Again we see that the overall PSF is different from the corresponding quasi-monochromatic PSF in Fig. 3. Often, the overall PSF of a system under white light is modeled as a two-dimensional Gaussian function (e.g. [10, 6, 9]). However, our two examples here shows that the overall PSF can be quite different from a two-dimensional Gaussian, and therefore Gaussian is not a satisfactory model.

8 Conclusion

We have examined the relation between the distance of an object from an image forming optical system, the parameters of the optical system, and the PSF of the optical system corresponding to the object. We have characterized the PSF in terms of a spread parameter defined as the standard deviation of the distribution of the PSF. An approximate equation where the spread paramter is linearly related to the reciprocal of the distance of the object has been derived. This equation has been verified through computer simulation. This suggests that the spread parameter is a useful measure of the distance of an object from a camera system. These results provide a theoretical basis for some of our earlier experimental work on finding the distance of objects from their blurred images. These results are also useful in machine vision applications and research.

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